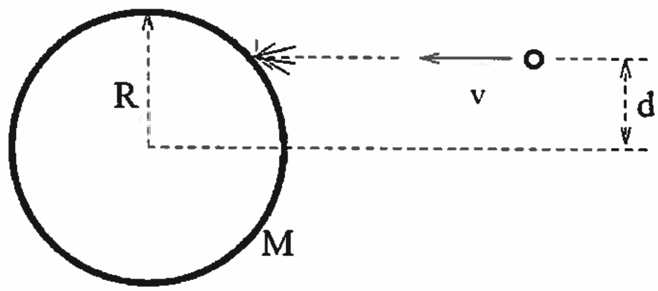
Practice problems before the Final Exam (II)

Problem Solving Activity **Name:**________________

In addition to solving these practice problems taken from ***the final exams given in previous years****, reviewing the past exams, the homework assignments and the workshop materials is also recommended.*

1. Phobos is a small moon of Mars. It has a mass of M=5.8 10^{15} kg and a radius of R=7.5 10^3 m. For the purpose of the following problem, assume that Phobos has the shape of a uniform sphere and that it is initially at rest. Suppose a meteorite strikes Phobos at distance d=5.0 10^3 m off center and embeds itself inside Phobos, close to its surface. If the meteorite mass was m=3.0 10^8 kg and its speed was v=1.5 10^5 m/s, what is the angular velocity ω of Phobos about its axis of rotation after the collision?



$L_i = L_f$

$m v d = I \omega_f$

$I = \underbrace{\frac{2}{5} M R^2}_{I_{phobos}} + \underbrace{m R^2}_{I_{meteorite}}$ (ok to neglect mR^2)

$= 1.3 \cdot 10^{23} + \underbrace{1.7 \cdot 10^{6}}_{\approx 0}$

$\omega_f = \frac{m v d}{I} = 1.7 \cdot 10^{-6} \frac{rad}{s} = 8.4 \frac{revolutions}{year}$

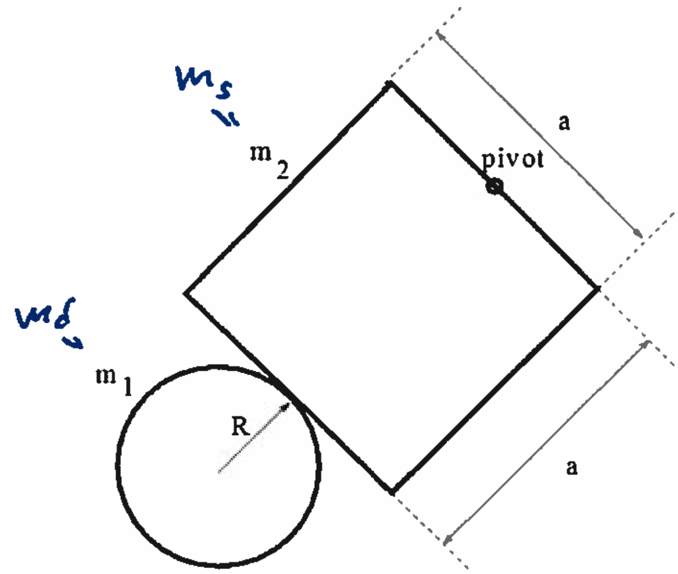
2. A harmonic transverse wave traveling on a cord has a wavelength of 0.1m, a frequency of 400 Hz, and an amplitude of 0.02m. What is the speed of the wave?

$\lambda = 0.1\,m$ $f = 400\,Hz$

$A = 0.02\,m$

$v = f \cdot \lambda = 400 \cdot 0.1 = 40\,\frac{m}{s}$

3. A rigid object consists of a uniform disk of mass m_1=3kg and radius R=4m attached to a uniform square of mass m_2=5kg and width a=10m. The object is suspended on a pin located at the center of one side of the square as shown below.



3a. Calculate the moment of inertia of the object (I) relative to the pin.

$$I = I_d + I_s = I_{d\,cm} + m_d(R+a)^2 + I_{s\,cm} + m_s\left(\frac{a}{2}\right)^2$$

$$= \frac{1}{2} m_d R^2 + m_d(R+a)^2 + \frac{1}{12} m_s (a^2+a^2) + m_s\left(\frac{a}{2}\right)^2$$

$$= \frac{1}{2} 3\cdot 4^2 + 3(4+10)^2 + \frac{1}{12}\cdot 5\cdot 2\cdot 10^2 + 5\cdot 5^2$$

$$= 24.0 + 588.0 + 83.3 + 125$$

$$= 820.3 \text{ kg}\cdot\text{m}^2$$

3b. Find the distance d between the center of mass of the object and the pin.

$$d = \frac{m_d (R+a) + m_s \frac{a}{2}}{m_d + m_s}$$

$$= \frac{3(4+10) + 5 \cdot \frac{10}{2}}{3+5}$$

$$= 8.375 \text{ m}$$

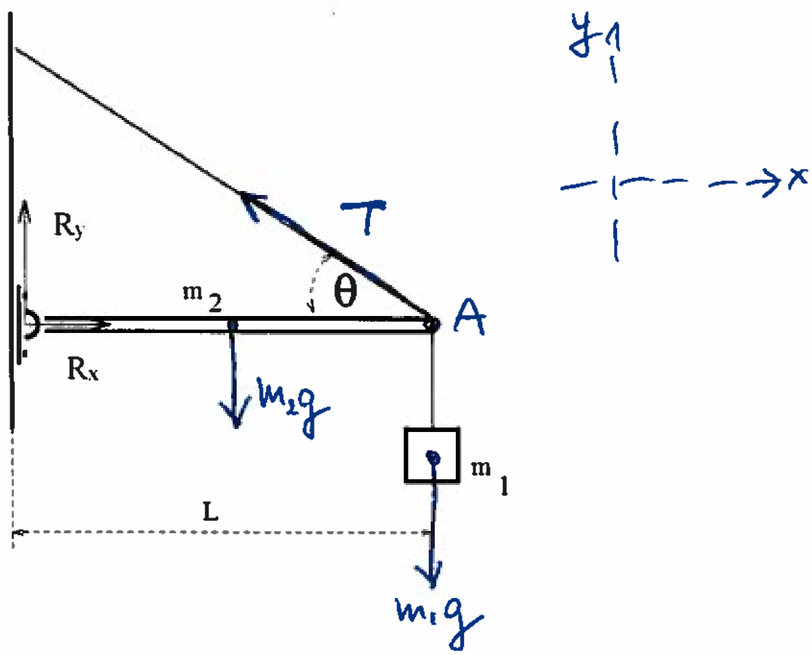
3c. Assuming that the object is released from rest when the center of mass in nearly straight down from the pin, what is the period of its oscillations? (If you have not solved 3a or 3b assume that that I and d are given)
(g=10 m/s^2)

$$T = 2\pi \sqrt{\frac{I}{d \cdot mg}}$$

$$= 2\pi \sqrt{\frac{820.3}{8.375 \cdot (3+5) \cdot 10}}$$

$$= 6.95 \text{ s}$$

4. A block of mass m_1=3kg is suspended from the end of a uniform horizontal beam of length L=7m and mass m_2=5kg pinned to the wall at the other end (i.e. it is attached to the wall using a hinge). The beam is suspended on a cable attached to its end creating an angle of θ=35° with the beam (see below). What are the horizontal (R_x) and vertical (R_y) components of the reaction force exerted by the pin on the beam? (g=9.8m/s²)



$\sum F_{ix} = 0$ $\quad R_x - T\cos\theta = 0$

$\sum F_{iy} = 0$ $\quad R_y + T\sin\theta - m_1g - m_2g = 0$

$\sum_i \tau_i = 0$ $\quad -R_y \cdot L + m_2 g \frac{L}{2} = 0$ } τ with respect to point A

$$R_y = \frac{1}{2} m_2 g = \frac{1}{2} 5 \cdot 9.8 = 25 \text{ N}$$

$R_x = T\cos\theta$

$T\sin\theta = (m_1 + m_2)g - R_y = \left(m_1 + \frac{m_2}{2}\right)g$

$$R_x = \left(m_1 + \frac{m_2}{2}\right)g \frac{\cos\theta}{\sin\theta} = \left(3 + \frac{5}{2}\right) \cdot 9.8 \cdot \frac{\cos 35^\circ}{\sin 35^\circ} = 77 \text{ N}$$

Table 9.2 Moments of Inertia of Various Bodies