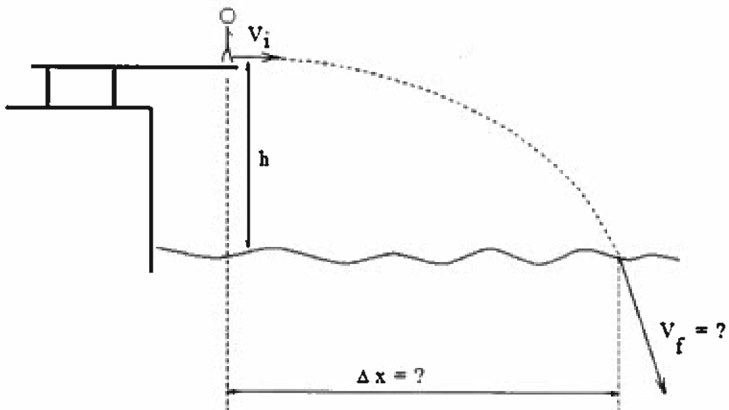
Practice problems before the Final Exam (I)

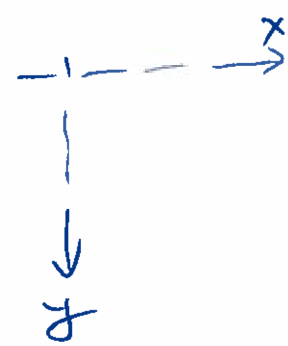
Problem Solving Activity **Name:**____________________

*In addition to solving these practice problems taken from **the final exams given in previous years**, reviewing the past exams, the homework assignments and the workshop materials is also recommended.*

1. A diver runs off the diving board located at h=2m above the water with initial velocity v_i=3 m/s directed horizontally.



1a. How far does she fly in horizontal direction, Δx, before entering the water? (g=10m/s^2)



$$\begin{cases} \Delta x = v_i \, \Delta t \\ h = \Delta y = \frac{1}{2} g \, \Delta t^2 \end{cases}$$

$$\Delta t = \sqrt{\frac{2h}{g}}$$

$$\Delta x = v_i \sqrt{\frac{2h}{g}} = 3\sqrt{\frac{2\cdot 2}{10}} = 3 \cdot 0.63 = 1.9\ m$$

1b. What is her speed (i.e. magnitude of total velocity) v_f when she enters the water? (g=10m/s^2)

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

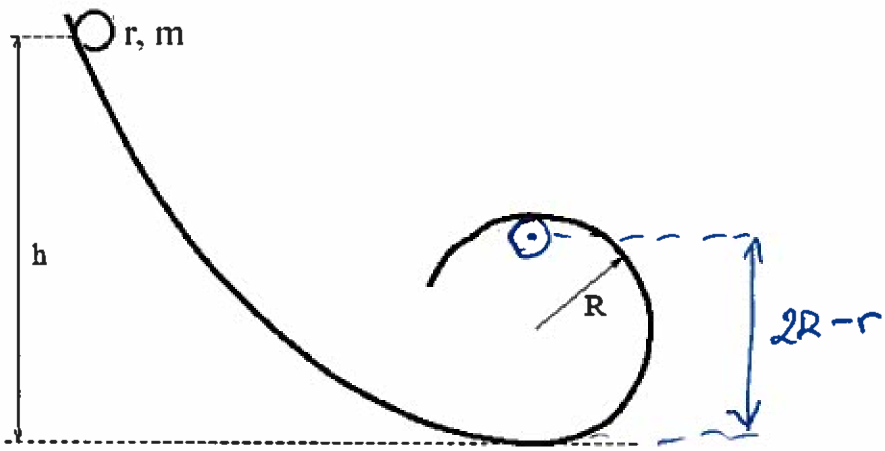
$$v_{fx} = v_i$$

$$v_{fy} = g\,\Delta t = g\sqrt{\frac{2h}{g}} = \sqrt{2hg}$$

$$\boxed{v_f = \sqrt{v_i^2 + 2hg}} = \sqrt{3^2 + 2\cdot 2\cdot 10} = 7.0\ \frac{m}{s}$$

(can also use energ conservation
$E_i = E_f$
$\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_f^2$
$v_f = \sqrt{v_i^2 + 2hg}$)

2. A uniform ball of mass r =0.1 m and mass m=3 kg rolls down without slipping along loop-the-loop track shown below. The radius of the loop is R=1.6 m. The ball is released from rest with its center at the height h = 12 m above the bottom of the track.



2a. What is the speed of center-of-mass of the ball at the top of the loop? (g=10m/s^2)

no slipping → mechanical energy is conserved

$$E_i = E_f$$

$$\underbrace{mgh}_{U_i} = \underbrace{\frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\,\omega^2}_{K_f} + \underbrace{mg(2R-r)}_{U_f}$$

$(K_i = 0)$

$$I_{CM} = \frac{2}{5}mr^2$$

$$\omega = \frac{v}{r}$$

$$gh = \frac{7}{10}v^2 + g(2R-r)$$

$$\boxed{v = \sqrt{\frac{10}{7}g(h-2R+r)}} = 11.3\ \frac{m}{s}$$

2b. What is the magnitude of the normal force exerted by the track on the ball at the top of the loop? ($g=10m/s^2$)

$$N + mg = F_{net} = m\frac{v^2}{r} \qquad r = R - r$$

$$N = m\left(\frac{v^2}{R-r} - g\right) = 224\ N$$

3. A bullet is shot through a wooden block. The bullet has a mass of 0.03kg and its initial speed is 800 m/s. The block is initially at rest and has a mass of 1.5kg. The block has a speed of 5 m/s right after the bullet went through. Calculate the speed of the bullet after it emerged from the block.

$m_1 = 0.03\ kg$ $\qquad$ $m_2 = 1.5\ kg$

i :

$v_{1i} = 800\ \frac{m}{s}$

$v_{2i} = 0$

f:

$v_{2f} = 5\frac{m}{s}$ $\qquad$ $v_{1f} = ?$

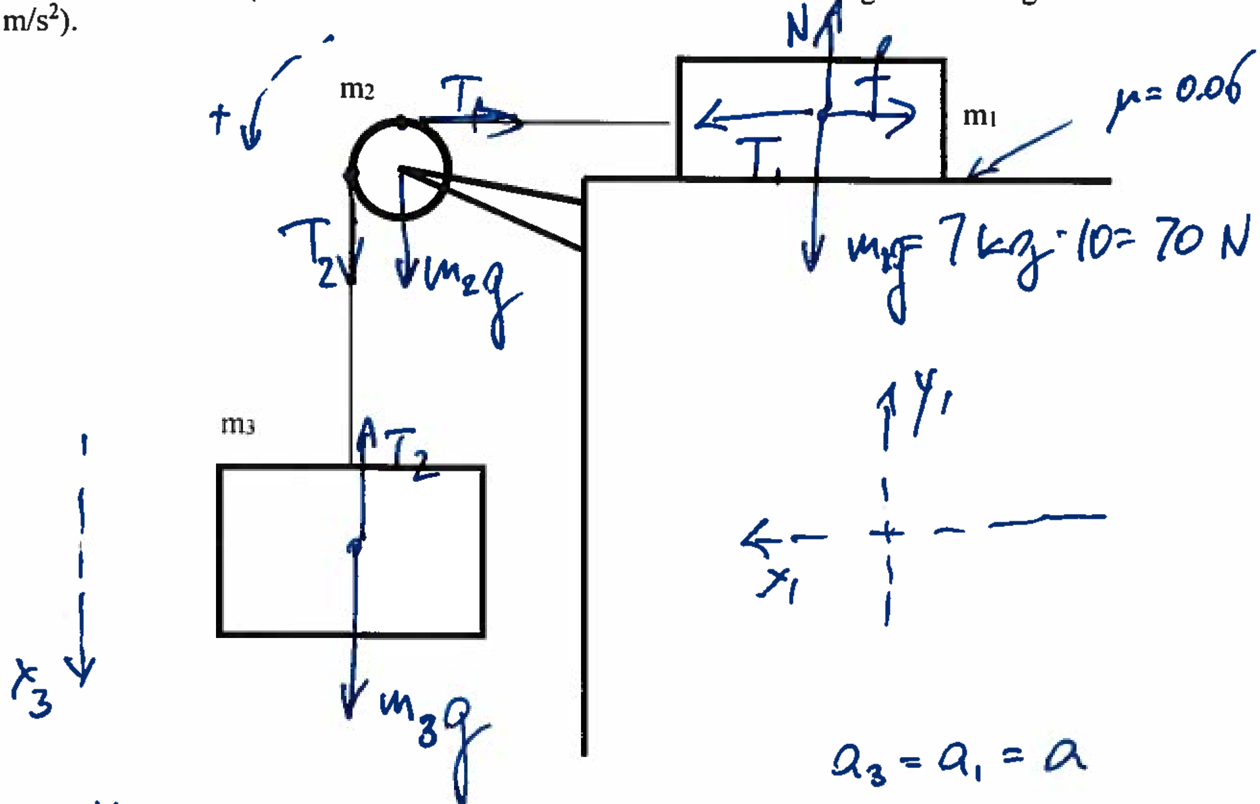
momentum conservation

$$P_i = P_f$$

$$m_1 v_{1i} + m_2 \underset{0}{\underset{\parallel}{v_{2i}}} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = v_{1i} - \frac{m_2}{m_1} v_{2f} = 550\ \frac{m}{s}$$

4. Two blocks with masses m_1=7 kg and m_3=5 kg are connected by a massless string via pulley with a mass of m_2=6 kg. Assume the pulley is a uniform disk and that it rotates without a friction on its axle. The string is non-stretchable and doesn't slip on the pulley. Coefficient of kinetic friction for the block on the horizontal surface is μ=0.06. Find acceleration of this block assuming it is moving to the left (use g=10 m/s^2).



$a_3 = a_1 = a$

$$\begin{cases} 0 = N - m_1 g \\ m_1 a = T_1 - f \\ f = \mu N \\ I_2 \alpha = R_2 T_2 - R_2 T_1 \\ \tfrac{1}{2} m_2 R_2^2 \tfrac{a}{R_2} \\ m_3 a = m_3 g - T_2 \end{cases} \rightarrow \begin{cases} m_1 a = T_1 - \mu m_1 g \\ \tfrac{1}{2} m_2 a = T_2 - T_1 \\ m_3 a = m_3 g - T_2 \end{cases}$$

$$(m_1 + \tfrac{1}{2} m_2 + m_3) a = m_3 g - \mu m_1 g$$

$$\boxed{a = \frac{m_3 - \mu m_1}{m_1 + \frac{1}{2} m_2 + m_3} g} = \frac{5 - 0.06 \cdot 7}{7 + \frac{6}{2} + 5} 10 = 3.05 \frac{m}{s^2}$$

Table 9.2 Moments of Inertia of Various Bodies