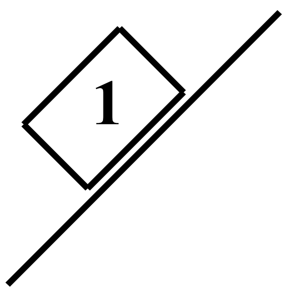
# Welcome back to Physics 211

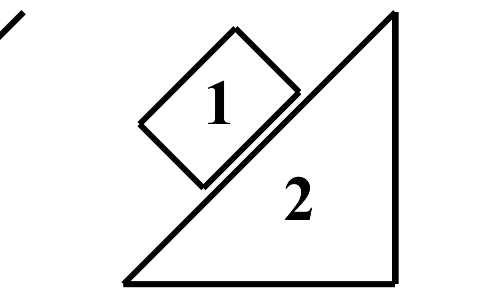
Today's agenda:

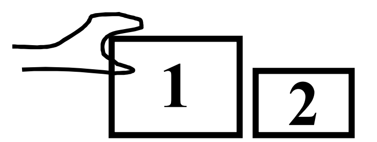
- Applying Newton's Laws

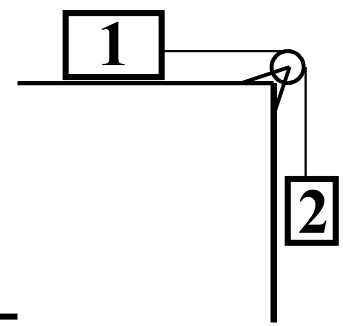
# Suggested procedure for problem solving using Newton's laws

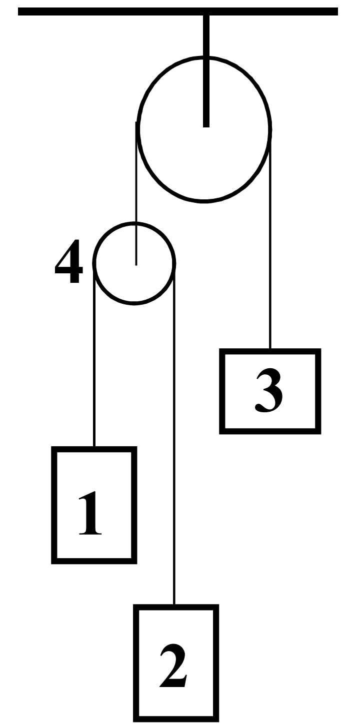
1. Identify moveable parts of the system:
    - Usually need to treat them separately









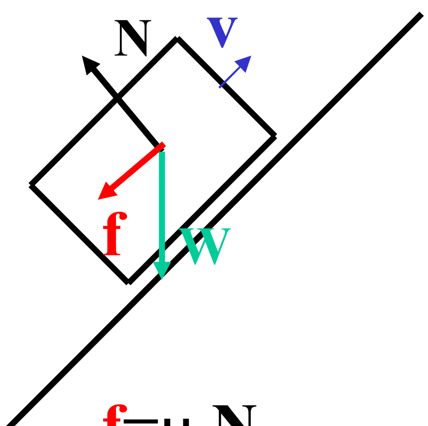


# 2. Draw all forces on each moveable part:

- Weight (W=mg)
- Normal, Tension, static friction (unknown)
  - > If **maximal** static friction: $f_s = \mu_s N$
- Kinetic friction ($f_k = \mu_k N$)
- Any additional forces (e.g. push by hand)
- **Make sure Newton's 3rd law is satisfied**

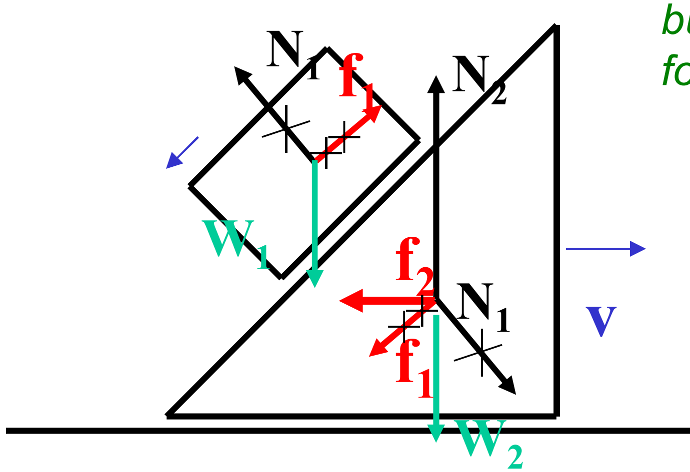
*Must know direction of velocity for direction of kinetic friction.*
*Wrong direction of frictional force will produce wrong results.*

*In case of static friction, must guess in which direction parts would move.*
*Wrong guess does not lead to errors, but derived magnitude of frictional forces will be negative.*



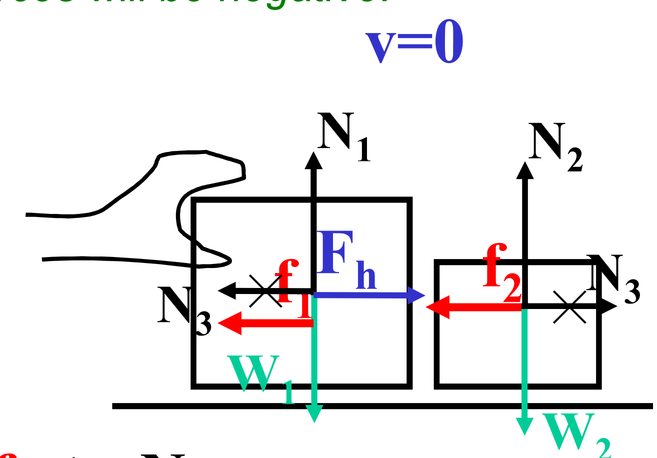

$f = \mu N$

$W = m\,g$

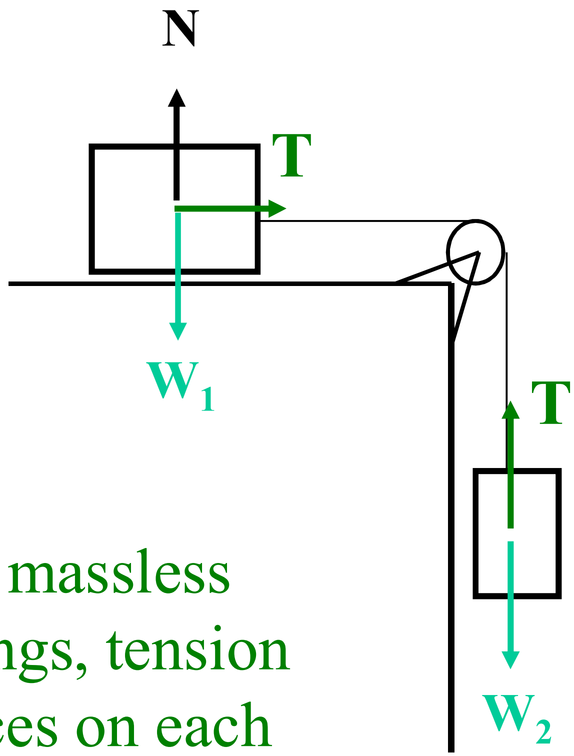



$f_1 = \mu_1 N_1$ $\quad W_1 = m_1\,g$

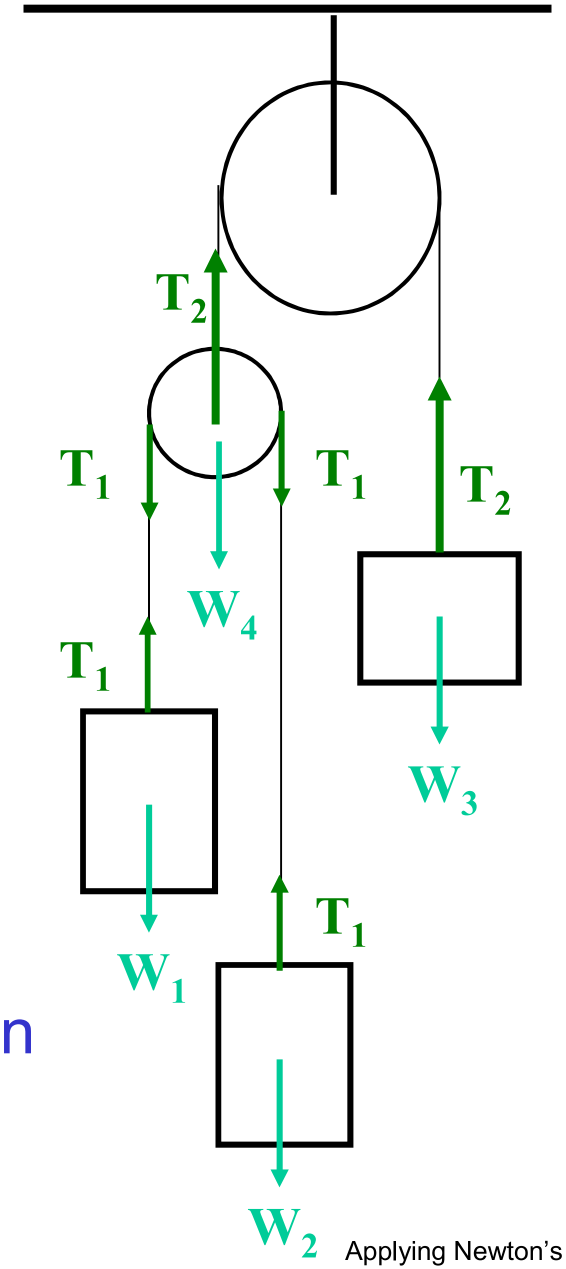
$f_2 = \mu_2 N_2$ $\quad W_2 = m_2\,g$




$f_i \leq \mu_i N_i$ usually not useful !



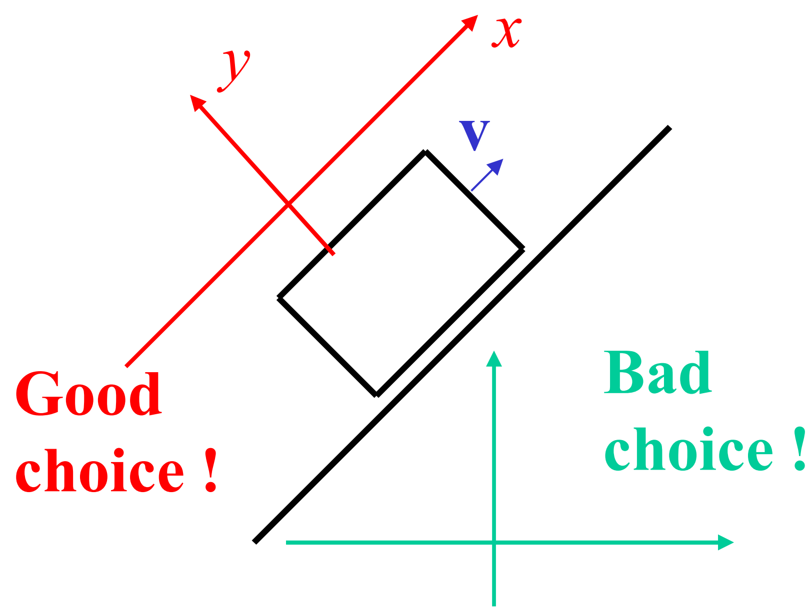

For massless strings, tension forces on each end of the string are the same



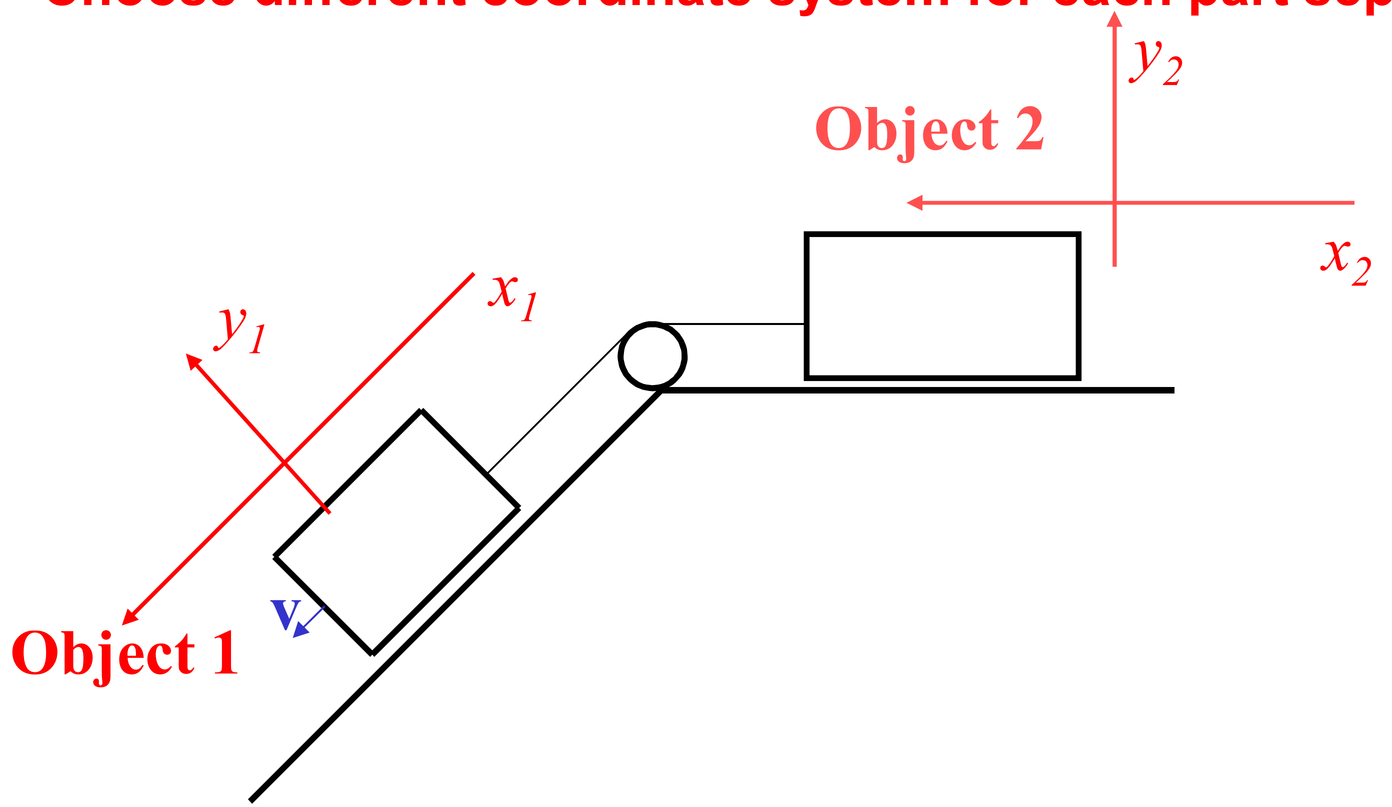

Each piece of String has its own Tension ($T_1$, $T_2$)

# 3. Choose coordinate system for each moveable part

- In almost all problems parts move along straight lines
  - **It pays off to choose one axis along the direction of motion!**



- If different parts move in different directions
  - **Choose different coordinate system for each part separately**



# 4. Write Newton's 2nd law for each part of the system

$$m\,\vec{a} = \sum_i \vec{F}_i$$

$$\Updownarrow$$

$$\begin{cases} m\,a_x = \sum_i F_{ix} \\ m\,a_y = \sum_i F_{iy} \end{cases}$$

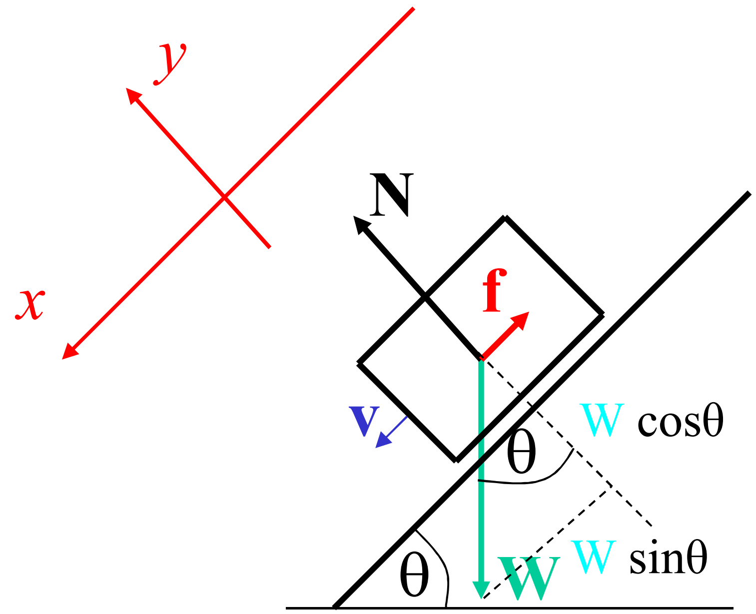
2 equations per part

- In usual case the part moves along only one reference axis (assume $x$)

$$v_y = 0 \quad \Rightarrow \quad a_y = 0$$

Can also drop subscript $x$ from $a_x$, since the total acceleration in this direction

$$\begin{cases} m\,a = \sum_i F_{ix} \\ 0 = \sum_i F_{iy} \end{cases}$$




$$\begin{cases} m\,a_x = W_x + N_x + f_x = W\sin\theta + 0 - f \\ m\,a_y = W_y + N_y + f_y = -W\cos\theta + N + 0 \end{cases}$$

$$\begin{cases} m\,a = W\sin\theta - f \\ 0 = -W\cos\theta + N \end{cases}$$

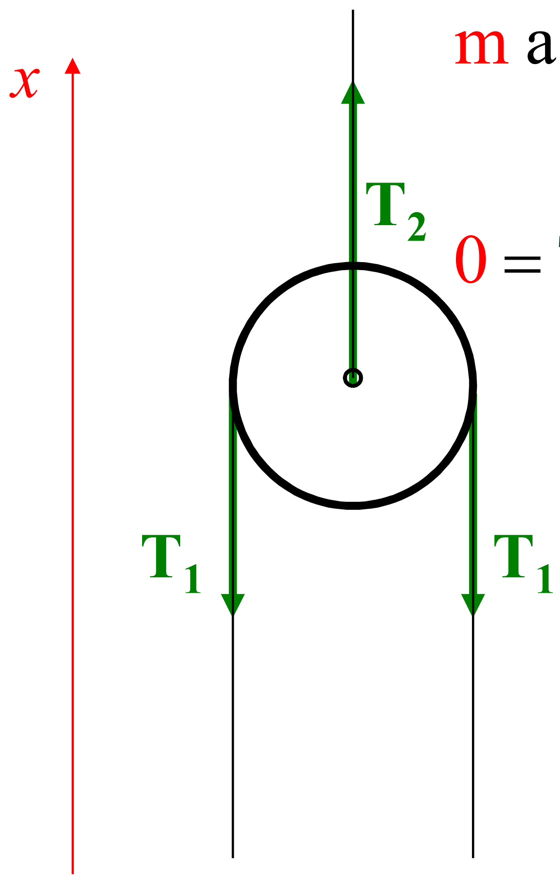
(will also likely need $f = \mu_k N$)

Pay attention to the sign of the force components!

# Be aware of massless objects

## $m=0 \mapsto F_{net}=0$ even if $a\neq 0$

**Massless pulley**

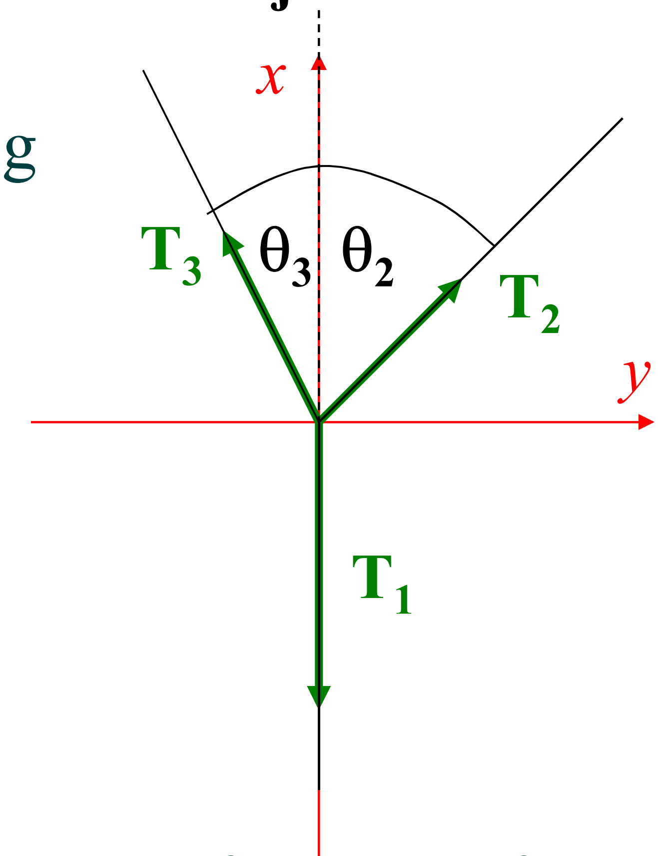



$$m\,a_x = T_2 - T_1 - T_1 - m\,g$$

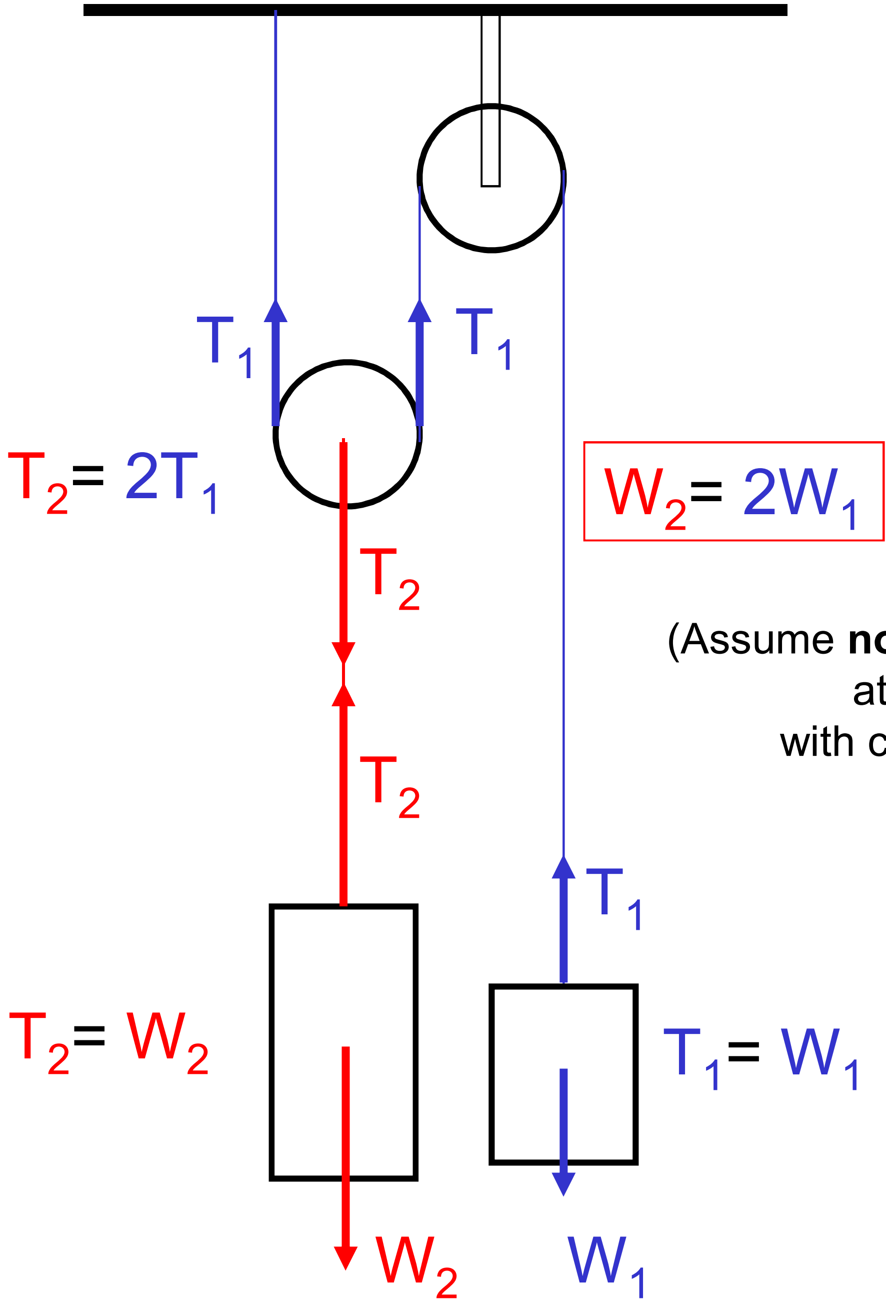
$$\Downarrow \quad m = 0$$

$$0 = T_2 - 2T_1$$

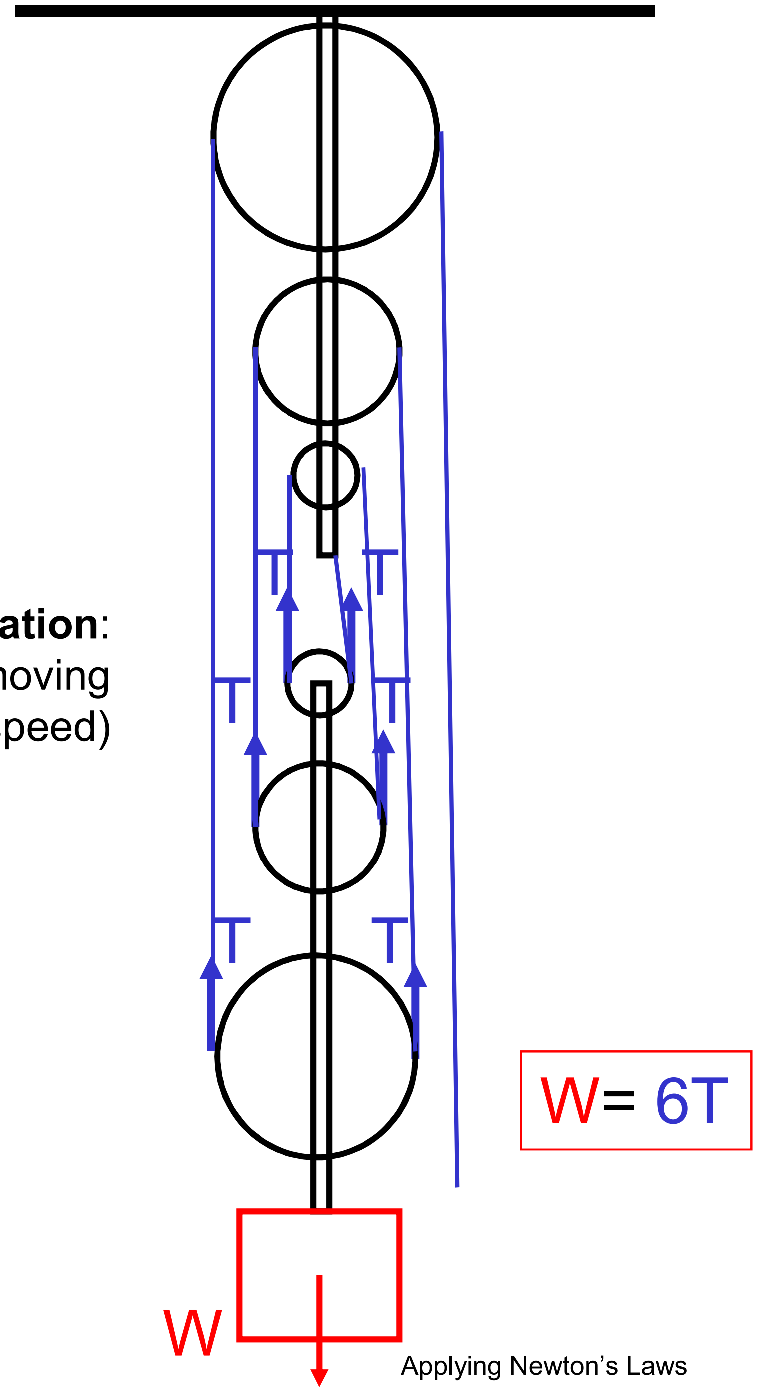
**Cord junction**




$$\begin{cases} 0 = T_2\cos\theta_2 + T_3\cos\theta_3 - T_1 \\ 0 = T_2\sin\theta_2 - T_3\sin\theta_3 \end{cases}$$



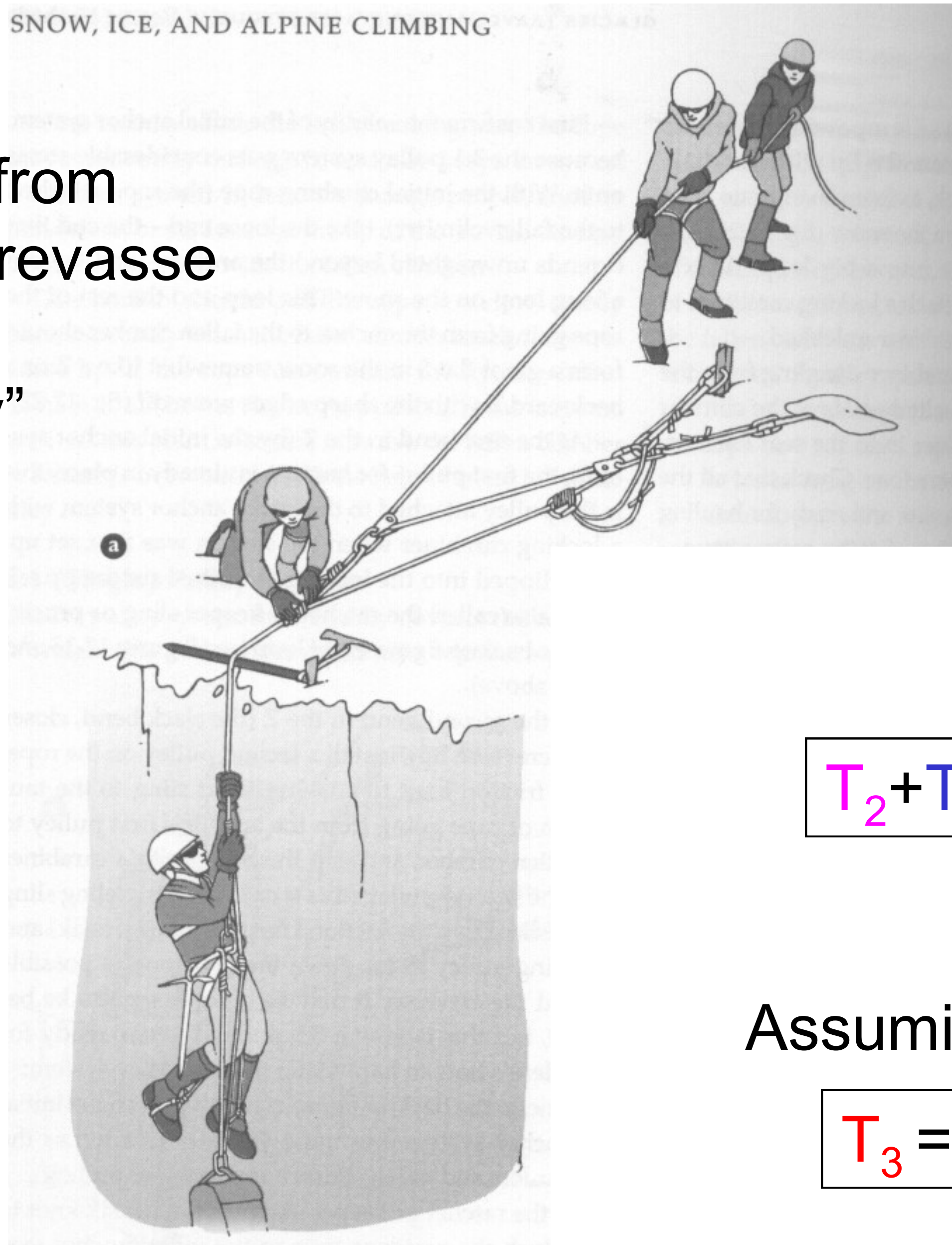

(Assume **no acceleration**:
at rest or moving
with constant speed)

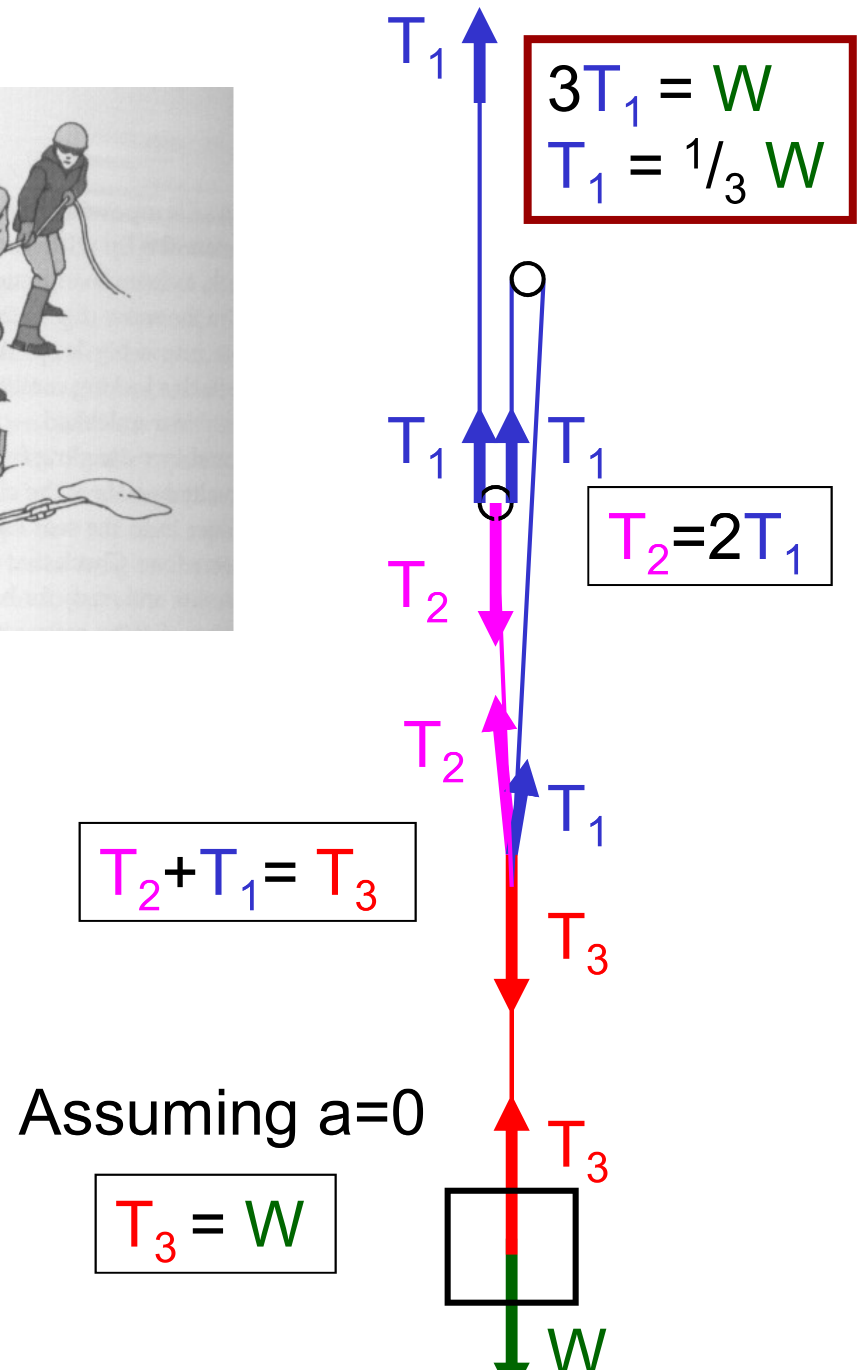


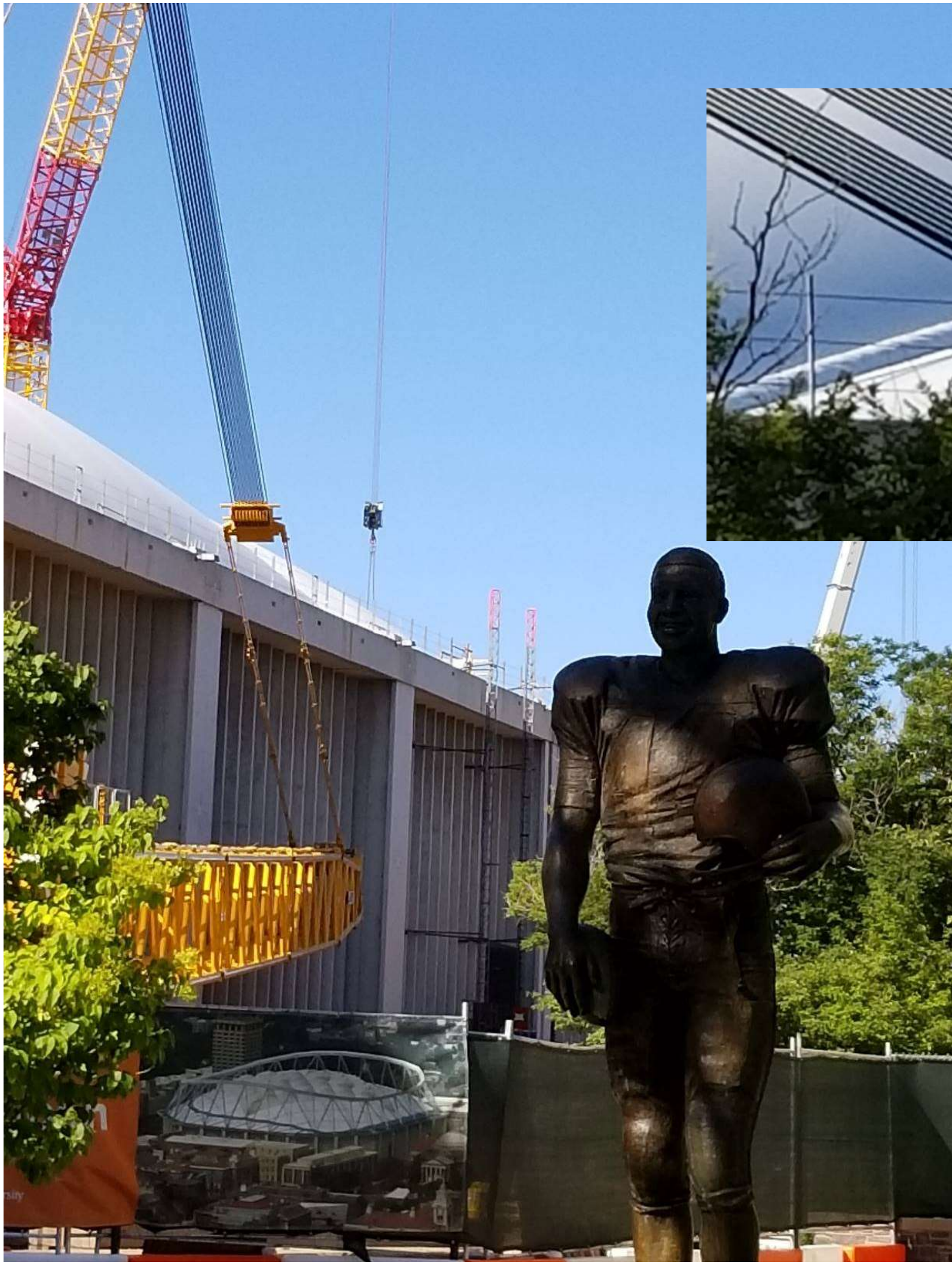
glacier crevasse

Rescue from glacier crevasse

"Z-pulley"

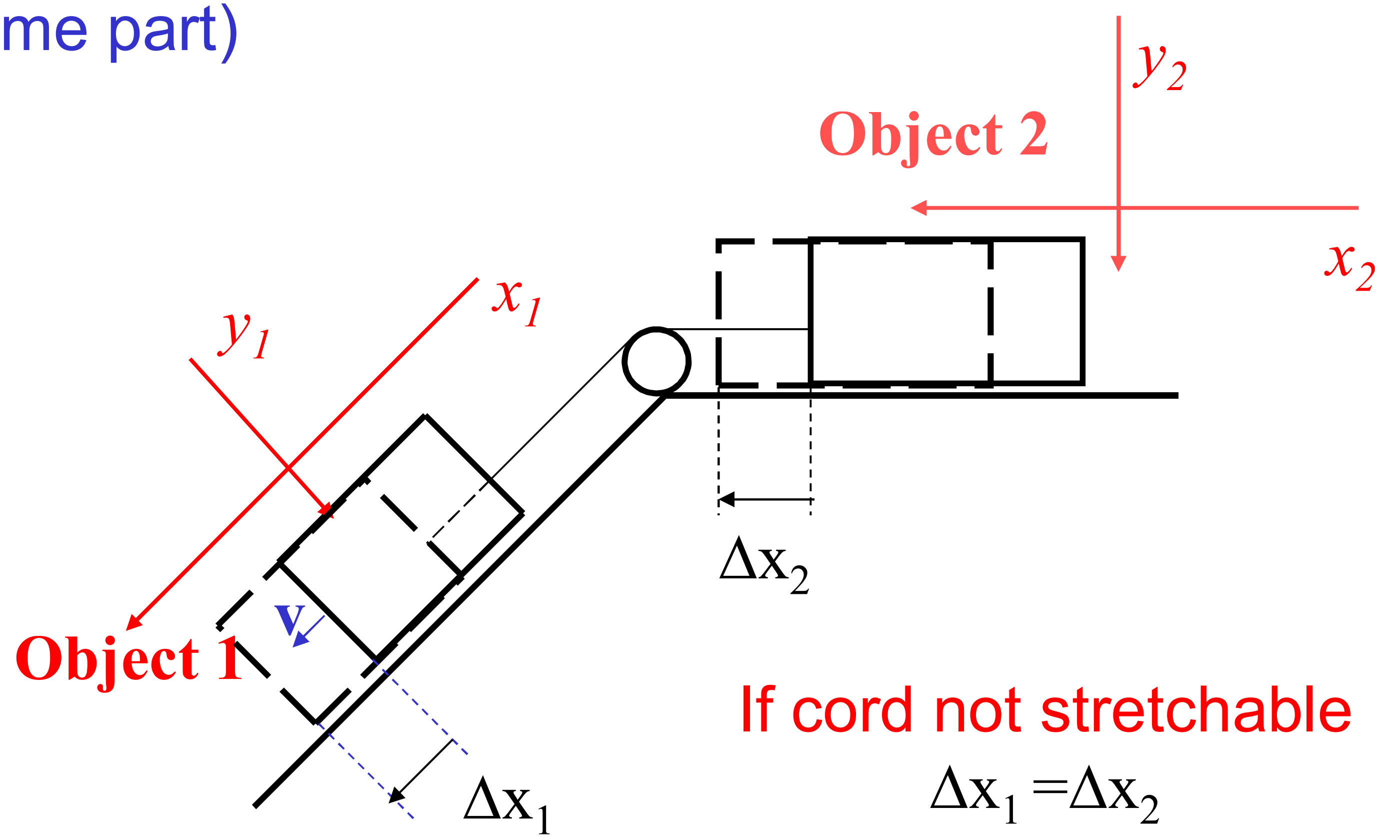




Same cable tension applied 24x to the crane arm !

# 5. Write all constraints among accelerations of different parts (or different components of accelerations for the same part)




If cord not stretchable

$$\Delta x_1 = \Delta x_2$$

$$v_1 = v_2$$

$$a_1 = a_2$$

## 6. See if the set of equations is solvable

- Number of unknown variables should be equal (or less) than number of independent equations
- Otherwise look for missed relations or additional constraints

## 7. Solve the system of equations

- Solve the system of equations for the unknown(s) of interest
  - Eliminate unknowns that you are not interested in

# Suggested procedure for problem solving using Newton's laws

1. Identify moveable parts of the system
2. Draw all forces on each moveable part
3. Choose coordinate system for each moveable part
4. Write Newton's $2^{nd}$ law for each part of the system
5. Write all constraints among accelerations of different parts (or different components of accelerations for the same part)
6. See if the set of equations is solvable (any additional equations?)
7. Solve the system of equations

Your MP homework "Applying Newton's $2^{nd}$ Law – 2 blocks" outlines this procedure plus additional steps to check validity of the solution:

8. Check dimensions (units) of your answer
9. Check if your answer satisfies special cases (requires symbolic solution)

A block with mass $m_1$=3kg is placed on an inclined plane with slope $\alpha$=20° and is connected to a second block with mass $m_2$=5kg by a massless not stretchable cord passing over a massless, frictionless pulley. The coefficient of kinetic friction is $\mu$=0.2. Assuming the second block is going down, find acceleration of the first block.

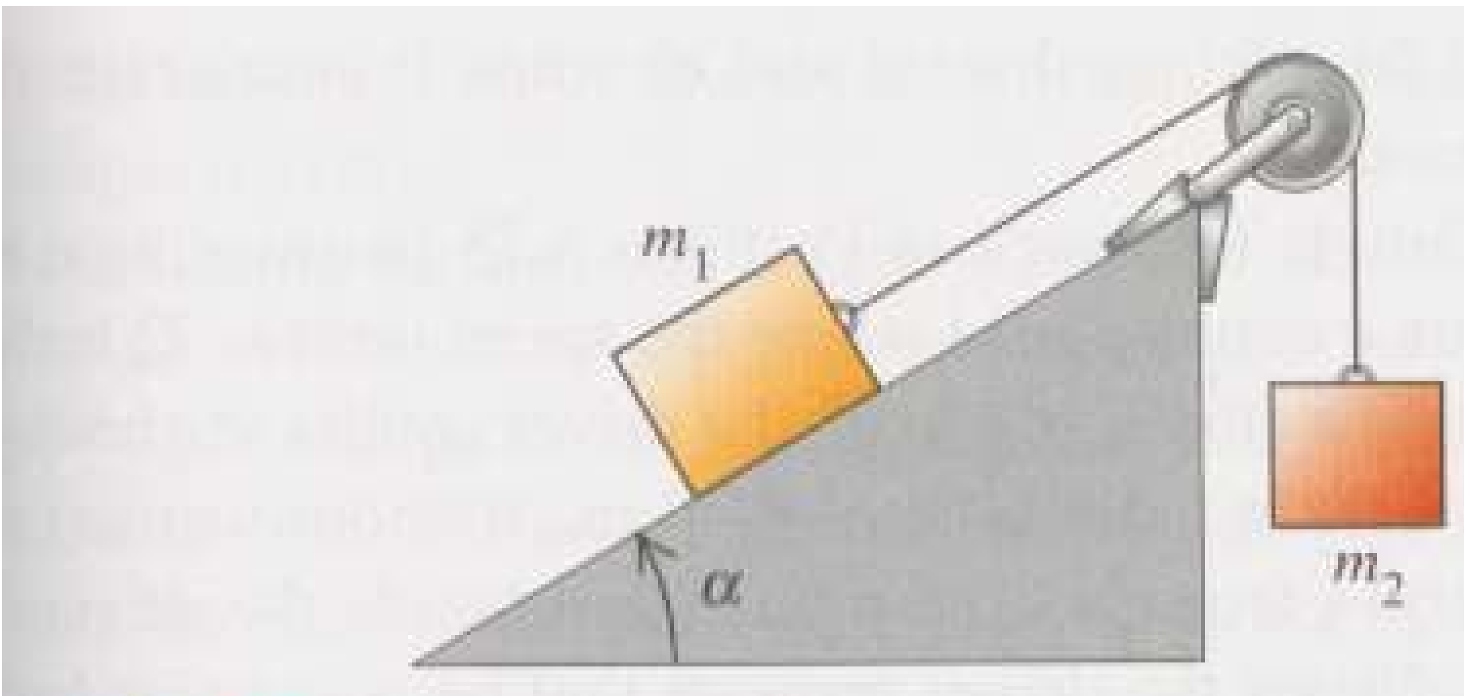
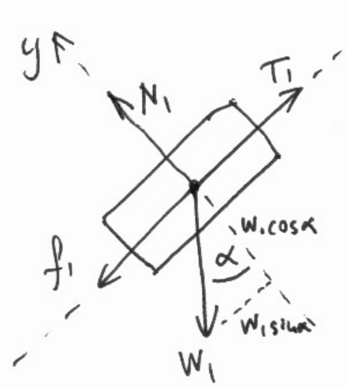
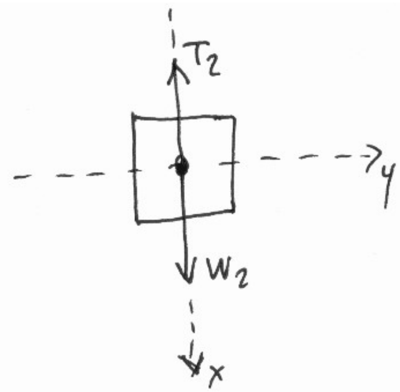



Figure 5.56 Problem 5.60.




physics

Newton's 2nd law:

$$m_1 a_{1x} = W_{1x} + T_{1x} + N_{1x} + f_{1x}$$
$$W_{1x} = -m_1 g \sin\alpha, \quad T_{1x} = T_1, \quad N_{1x} = 0, \quad f_{1x} = -f_1$$

$$m_1 a_{1y} = W_{1y} + T_{1y} + N_{1y} + f_{1y}$$
$$m_1 a_{1y} = 0, \quad W_{1y} = -m_1 g \cos\alpha, \quad T_{1y} = 0, \quad N_{1y} = N_1, \quad f_{1y} = 0$$

$$m_2 a_{2x} = W_{2x} + T_{2x}$$
$$W_{2x} = m_2 g, \quad T_{2x} = -T_2$$

$$m_2 a_{2y} = W_{2y} + T_{2y}$$
$$m_2 a_{2y} = 0, \quad W_{2y} = 0, \quad T_{2y} = 0$$

massless frictionless pulley

$$T_1 = T_2 \quad (= T)$$

not stretchable cord

$$a_{1x} = a_{2x} \quad (= a)$$

$$\begin{cases} m_1 a = -m_1 g \sin\alpha + T - f_1 \\ 0 = -m_1 g \cos\alpha + N_1 \\ m_2 a = m_2 g - T \\ f_1 = \mu N_1 \end{cases}$$

4 equations
4 unknowns

$a, T, f_1, N_1$

↓

solveable

## *using numbers*

(math)

unknowns: $a$, $T_1$, $f_1$, $N_1$

$$\begin{cases} 3a = -3\cdot 9.8\cdot \sin 20^\circ + T - f_1 \\ 0 = -3\cdot 9.8\,\cos 20^\circ + N_1 \\ 5a = 5\cdot 9.8 - T \\ f_1 = \mu N_1 \end{cases}$$

$$N_1 = 3\cdot 9.8\cdot \cos 20^\circ = 27.6\ \text{N}$$

$$f_1 = \mu N_1 = 0.2\cdot 27.6 = 5.53\ \text{N}$$

$$3a = -10.06 + T - 5.53$$

unknown: $a, T$

$$+\begin{cases} 3a = T - 15.6 \\ 5a = 49 - T \end{cases}$$

$$(3+5)a = T - 15.6 + 49 - T = -15.6 + 49 = 33.4$$

$$a = \frac{33.4}{8} = 4.2\ \frac{m}{s^2}$$

## *using symbols*

(math)

$$\begin{cases} m_1 a = -m_1 g \sin\alpha + T - f_1 \\ 0 = -m_1 g \cos\alpha + N_1 \\ m_2 a = m_2 g - T \\ f_1 = \mu N_1 \end{cases}$$

$N_1 = m_1 g \cos\alpha$

$f_1 = \mu m_1 g \cos\alpha$

$$T = m_2 g - m_2 a$$

$$m_1 a = -m_1 g \sin\alpha + m_2 g - m_2 a - \mu m_1 g \cos\alpha$$

$$m_1 a + m_2 a = m_2 g - m_1 g(\sin\alpha + \mu\cos\alpha)$$

$$(m_1 + m_2) a = [m_2 - m_1(\sin\alpha + \mu\cos\alpha)] g$$

$$\boxed{a = \frac{m_2 - m_1(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} g}$$

$$a = \frac{5 - 3(\sin 20^\circ + 0.2\cos 20^\circ)}{3+5} 9.8 = 4.2 \frac{m}{s^2}$$

$$a = \frac{m_2 - m_1(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} g$$

Power of symbolic solution!

Offers many ways to check if the solution is correct:

**Consistency of units:**

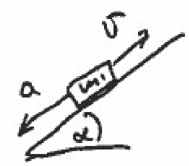
$a$ in $\frac{kg - kg\,(1+1\cdot 1)}{kg + kg}\frac{m}{s^2} = \frac{kg}{kg}\frac{m}{s^2} = \frac{m}{s^2}$ correct!

**Extreme situations in which correct answer is easy to guess:**

$m_1 = 0$ $\quad a = \frac{m_2 - 0}{0 + m_2} g = \frac{m_2}{m_2} g = g$

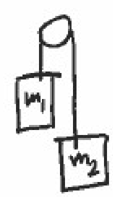

free fall correct!

$m_2 = 0$ $\quad a = \frac{0 - m_1(\sin\alpha + \mu\cos\alpha)}{m_1 + 0} g = -(\sin\alpha + \mu\cos\alpha) g$




single block on incline with friction going up

$\alpha = 90^\circ$ $\quad a = \frac{m_2 - m_1(1 + \mu\cdot 0)}{m_1 + m_2} g = \frac{m_2 - m_1}{m_1 + m_2} g$




Atwood's machine

$m_1 = m_2 \rightarrow a = 0\cdot g = 0$ correct!

$\alpha = 0^\circ$ $\quad a = \frac{m_2 - m_1(0 + \mu\cdot 1)}{m_1 + m_2} g = \frac{m_2 - \mu m_1}{m_1 + m_2} g$