Name (please print): ______________________________

Last *First*

SUID ____________________

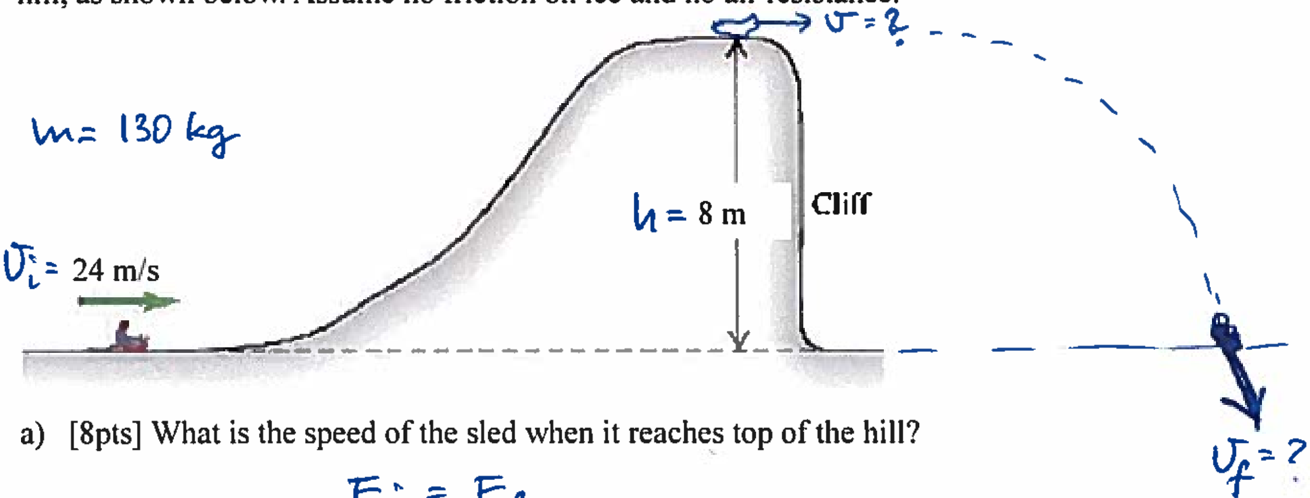
Physics 211, Fall 2019 Final Exam

Document your work or earn no credit. Use the back of each sheet if you run out of space. Cross over any parts that correspond to given up thoughts. ***For numerical answers give at least 2 significant digits and specify units.***

Assume g=10 m/s² throughout this exam.

1. [20 pts total] A sled with rider having combined mass of 130 kg travels over perfectly smooth icy hill, as shown below. Assume no friction on ice and no air resistance.

$m = 130\ kg$



a) [8pts] What is the speed of the sled when it reaches top of the hill?

$$E_i = E_f$$
$$K_i = K_f + U_f$$
$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + mgh$$
$$\boxed{v = \sqrt{v_i^2 - 2gh}} = \sqrt{24^2 - 2\cdot 10 \cdot 8} = \sqrt{576-160} = \sqrt{416}$$
$$= 20.4\ \frac{m}{s}$$

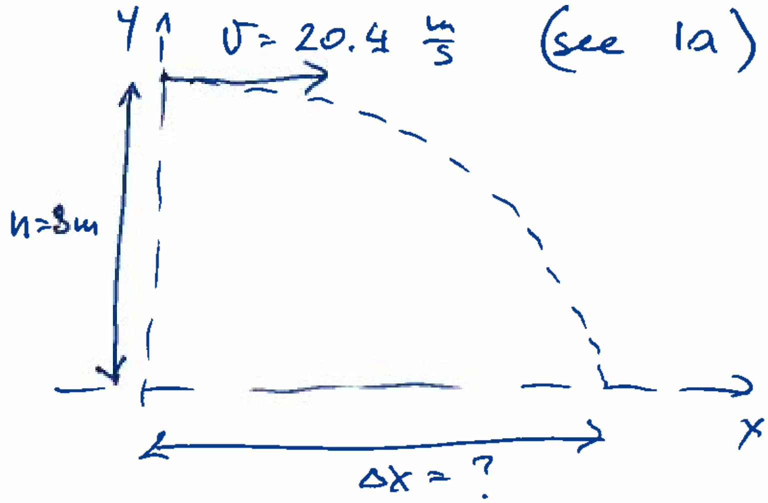
b) [2pts] What is the speed of the sled just before it lands on the ground after going horizontally off the cliff?

$$E_i = E_f$$
$$K_i = K_f$$
$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2$$
$$\boxed{v_f = v_i} = 24\ \frac{m}{s}$$

(can also by solved from free fall equations (see 1c) and

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$
$$v_{fx} = v = 20.4\ \frac{m}{s}$$
$$v_{fy} = -\frac{1}{2}g\Delta t^2$$
)

c) [10pts] How far does the sled land from the foot of the cliff?

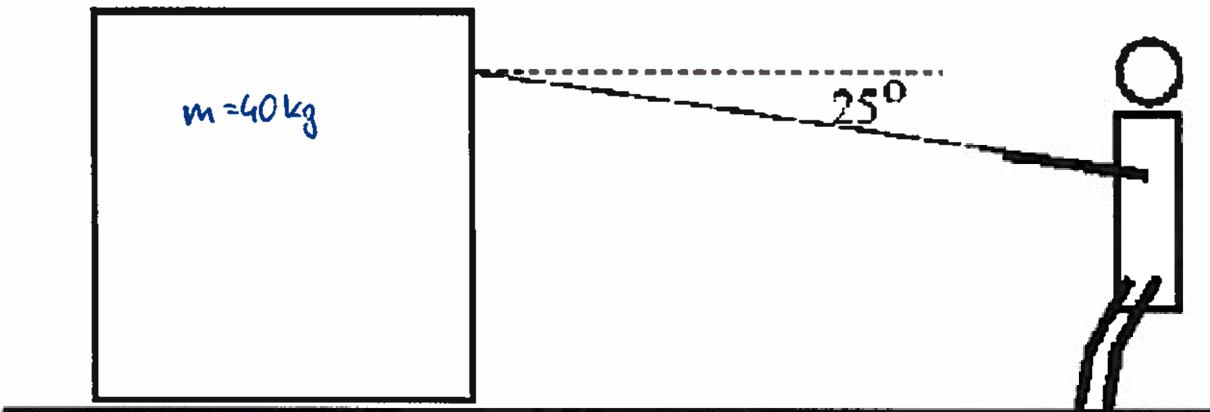


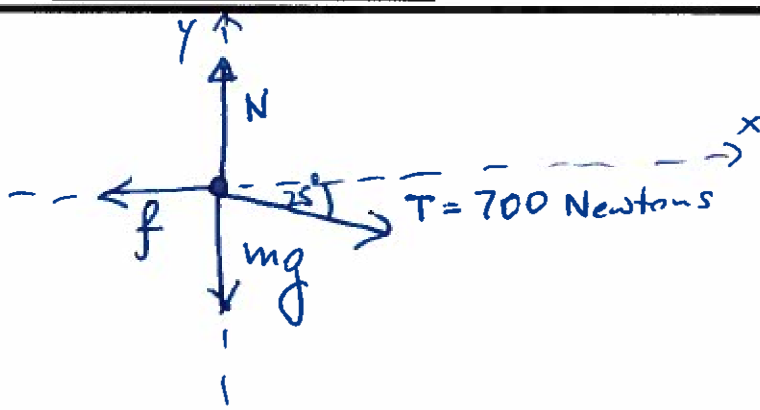
$$\begin{cases} \Delta x = v \cdot \Delta t \\ \Delta y = -h = -\frac{1}{2} g \Delta t^2 \end{cases}$$

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 8}{10}} = 1.26 \text{ s}$$

$$\Delta x = v \cdot \Delta t = 20.4 \cdot 1.26 = 25.8 \frac{m}{s}$$

2. [20pts] A person is dragging a crate by applying a force of 700 Newtons to the rope. The rope is attached to the crate as shown below. The crate has a mass of 40 kg. The coefficient of kinetic friction between the crate and the floor is μ=0.2. What is the acceleration of the crate? ($\cos 25^0$=0.9063, $\sin 25^0$=0.4226).





unknown
a, f, N

$$\begin{cases} ma = F_{net\,x} = T\cos\theta - f \\ 0 = F_{net\,y} = N - T\sin\theta - mg \\ f = \mu N \end{cases}$$

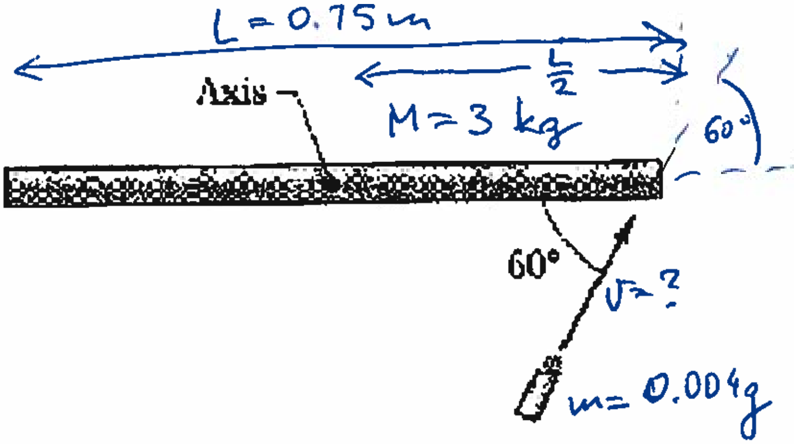
$$N = mg + T\sin\theta$$

$$ma = T\cos\theta - \mu(mg + T\sin\theta)$$

$$a = \frac{T}{m}(\cos\theta - \mu\sin\theta) - \mu g = \frac{700}{40}(0.9063 - 0.2\cdot 0.4226) - 0.2\cdot 10$$

$$= 12.4\ \frac{m}{s^2}$$

3. [15pts] A uniform thin rod of length 0.75 m and mass of 3.0 kg can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when 4.0 g bullet traveling in the horizontal plane of the rod is fired into one end of the rod. As viewed from above, the direction of the bullet's velocity makes an angle of 60^0 with the rod (see below). If the bullet lodges in the rod and the angular velocity is 10 rad/s immediately after the collision, what is the magnitude of the bullet's velocity just before impact? ($\cos 60^0 = 0.5000$, $\sin 60^0 = 0.8660$).



$$L_i = L_f$$

$$m v \frac{L}{2} \sin 60^\circ = I \omega_f$$

$$\omega_f = 10 \ \frac{rad}{s}$$

$$I = I_{rod} + m\left(\frac{L}{2}\right)^2$$

bullet in the rod

$$= \frac{1}{12} M L^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12} 3 \cdot 0.75^2 + 0.004\left(\frac{0.75}{2}\right)^2$$

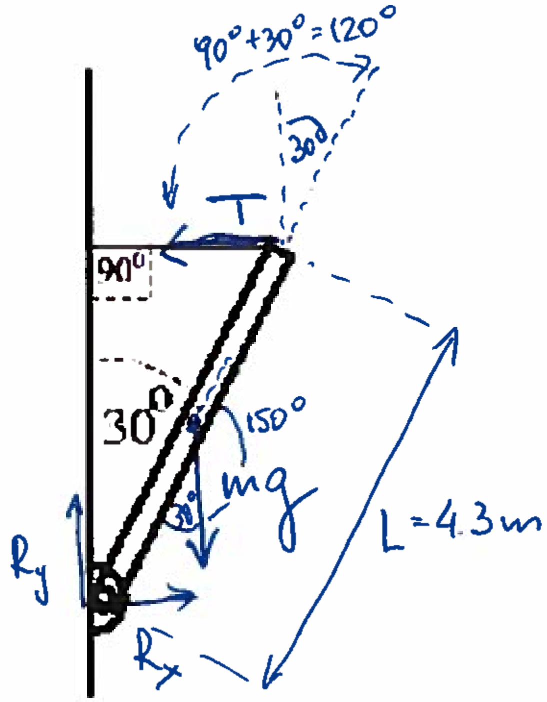
$$= 0.141 + 0.0006$$

$$= 0.141 \ kg \cdot m^2$$

OK to neglect

$$v = \frac{I \omega_f}{m \frac{L}{2} \sin 60^0} = \frac{0.141 \cdot 10}{0.004 \cdot \frac{0.75}{2} \cdot 0.866} = 1085 \ \frac{m}{s}$$

4. [15pts] A beam of length 4.3 m is hinged at the lower end and the upper end is held by a rope as shown below. The mass of the beam is 33 kg. What is the tension in the string?



torque with respect to the hinge:

$$0 = L \cdot T \cdot \sin 120° - \frac{L}{2} \cdot mg \cdot \sin 150°$$

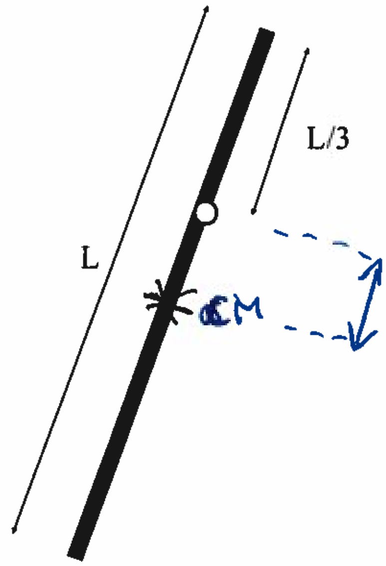
$$T = \frac{mg \sin 150°}{2 \cdot \sin 120°} = \frac{mg \sin 30°}{2 \cdot \sin 60°}$$

$$= \frac{33 \cdot 10 \cdot \frac{1}{2}}{2 \cdot 0.866}$$

$$= 95 \text{ Newtons}$$

$(R_x = T \quad R_y = mg)$

5. [15 pts] A slender rod of mass m=5 kg and length L=7 m is pivoted one-third of the length from the upper end as shown below. What is the period of its oscillations around its vertical orientation? (neglect friction at the pivot and air resistance)



$$d = \frac{L}{2} - \frac{L}{3} = L\left(\frac{3}{6} - \frac{2}{6}\right) = \frac{L}{6}$$

$$T = 2\pi \sqrt{\frac{I}{d \cdot mg}}$$

$$I = I_{CM} + md^2$$

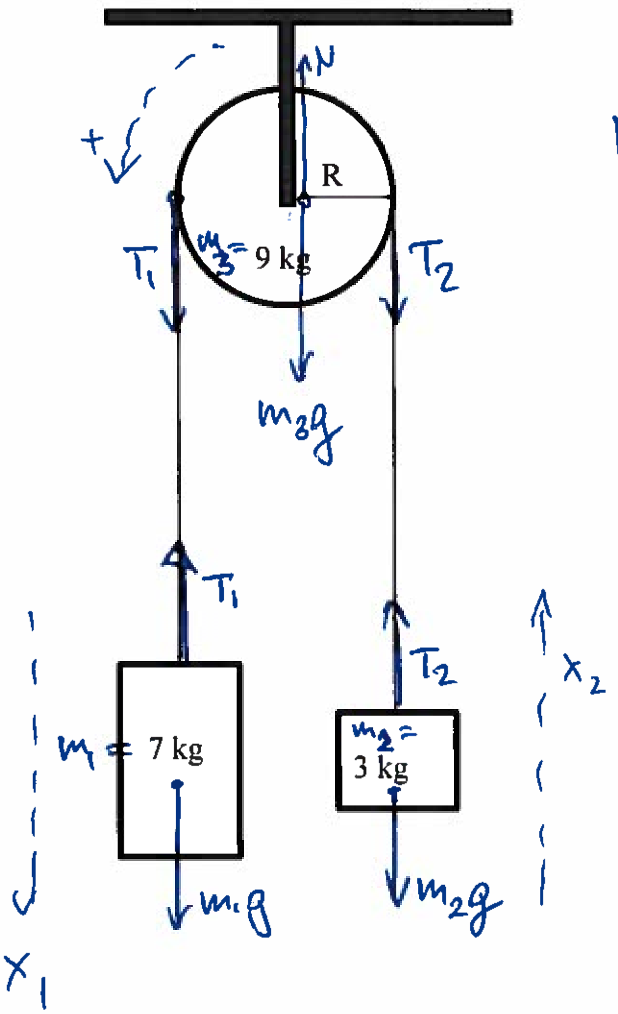
$$I_{CM} = \frac{1}{12} mL^2$$

$$T = 2\pi \sqrt{\frac{\frac{1}{12}\cancel{m}L^2 + \cancel{m}d^2}{d \cdot \cancel{m}g}} = 2\pi \sqrt{\frac{\frac{1}{12}L^2 + \frac{1}{36}L^2}{d \cdot g}}$$

$$= 2\pi \sqrt{\frac{\frac{4}{36}L^{\cancel{2}}}{\frac{\cancel{L}}{6} g}} = 2\pi \sqrt{\frac{\frac{1}{9}L}{\frac{1}{6}g}}$$

$$= 2\pi \sqrt{\frac{6}{9}\frac{L}{g}} = \underbrace{2\pi \sqrt{\frac{2}{3}\frac{L}{g}}} = 4.3\,s$$

6. [15pts] Two blocks with masses 7 and 3 kg are suspended on a string going around the pulley shown below. The mass of the pulley is 9 kg. Assume the pulley is a uniform disk of radius R=0.2 m, no friction on the pulley axel and non-stretchable massless string. What is the acceleration of the heavier block?



Newton's Law for blocks & pulley:

$$\begin{cases} m_1 a = m_1 g - T_1 \\ m_2 a = T_2 - m_2 g \\ \underset{\frac{1}{2} m_3 R^2}{I_3} \underset{\frac{a}{R}}{\alpha} = R T_1 - R T_2 \end{cases}$$

$$+\begin{cases} \frac{1}{2} m_3 a = T_1 - T_2 \\ m_1 a = m_1 g - T_1 \\ m_2 a = T_2 - m_2 g \end{cases}$$

$$\left(m_1 + m_2 + \tfrac{1}{2} m_3\right) a = m_1 g - m_2 g$$

$$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{m_3}{2}} g = \frac{7-3}{7+3+\frac{9}{2}} 10$$

$$= 2.76 \ \frac{m}{s^2}$$

Table 9.2 Moments of Inertia of Various Bodies