Review before final exam

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Guide how to identify type of the problem

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The question is about?

acceleration (linear or angular) | force | conditions for system at rest

Only if the problem explicitly says “average acceleration” or if the acceleration is constant $\mathbf{a}=D\mathbf{v}/D\mathbf{t}$ may be used

$a_x=0$

$a_y=0$

$\alpha=0$

The problem is for application of Newton’s 2nd Law:

Does/can center-of-mass of any object move?

$m\,a_x=\Sigma_i\,F_{i\,x}$

$(0=)\;m\,a_y=\Sigma_i\,F_{i\,y}$

Usually a_y is zero for proper choice of coordinates

Circular motion?

$a_x=v^2/R$

for the x-axis pointing towards the circle center

Does/can any object rotate?

$I\,\alpha=\Sigma_i\,\tau_i$

Also often needed:

$\alpha=a/R$

$\tau=\pm r\,F\sin\theta$

or $\pm r_{-}\;F$

Rolling combines both for the same object

The question is about?

velocity

Only if the problem explicitly says "average velocity" or if the velocity is constant $\mathbf{v}=D\mathbf{x}/D\mathbf{t}$ may be used

(linear or angular)

Wave velocity?

$v=\omega/k=f\,\lambda$

A free fall problem?

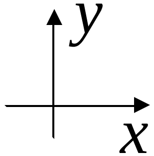
(the ***only*** force is weight)

$v_{fx}=v_{ix}$

$v_{fy}=v_{iy}-g\,\Delta t$

$\Delta x=v_{ix}\Delta t$

$\Delta y=v_{iy}\Delta t-{}^{1}/_{2}\,g\,(\Delta t)^2$



Some free fall problems are easier to solve using energy conservation

Collision?

(two objects, there is "before" and "after" the "interaction")

Any rotation involved?

yes

no

Use conservation of angular momentum

$L_{tot\ i}=L_{tot\ f}$

Extended object: $L=I\omega$

Point-like object:

$L=\pm\, r\, mv\sin\theta$

or $\pm\, r_{-}\ mv$

Use conservation of linear momentum

$P_{tot\ i}=P_{tot\ f}$

$p=mv$

Does the text say "elastic" ?

yes

In addition, use $K_i=K_f$

no

Does the text say "perfectly" inelastic or the objects stick to each other ?

yes

$v_{1f}=v_{2f}$

Use conservation of mechanical energy

$E_{tot\ i}=E_{tot\ f}$

$K_i+U_i=K_f+U_f$

Rotating object: $K={}^{1}/_{2}I\omega^2$

Linear motion: $K={}^{1}/_{2}mv^2$

Gravitational: $U=mgh$

Elastic (spring) : $U={}^{1}/_{2}kx^2$

The question is about? position

(linear or angular)

A free fall problem?

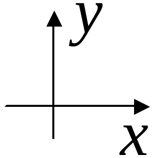
(the ***only*** force is weight)

$\Delta x = v_{ix}\Delta t$

$\Delta y = v_{iy}\Delta t - {}^1/_2\, g\,(\Delta t)^2$

$v_{fx} = v_{ix}$

$v_{fy} = v_{iy} - g\,\Delta t$



Some free fall problems are easier to solve using energy conservation

Use conservation of mechanical energy

$$E_{tot\ i} = E_{tot\ f}$$

$$K_i + U_i = K_f + U_f$$

Rotating object: $K = {}^1/_2 I\omega^2$

Linear motion: $K = {}^1/_2 mv^2$

Gravitational: $U = mgh$

Elastic (spring) : $U = {}^1/_2 kx^2$

Is velocity constant?

$$\Delta x = v\,\Delta t$$

Is acceleration constant?

$$\Delta x = v_i\,\Delta t + {}^1/_2\, a\,(\Delta t)^2$$

$$v_f = v_i + a\,\Delta t$$

linear	$\mapsto$	angular
x		θ
v	$\mapsto$	ω
a		α

Modification of the slide on “velocity” and “position” problems

…

Is mechanical energy conserved?

(Is work by external or non-conservative forces zero?)

yes

no

Use conservation of mechanical energy

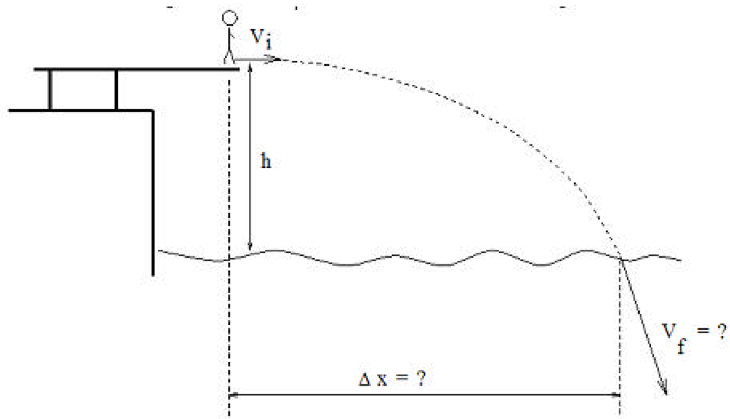
$$E_{tot\ i} = E_{tot\ f}$$

Use energy-work theorem

$$\Delta E_{tot} = W_{ext.\ or\ non\text{-}cons.}$$

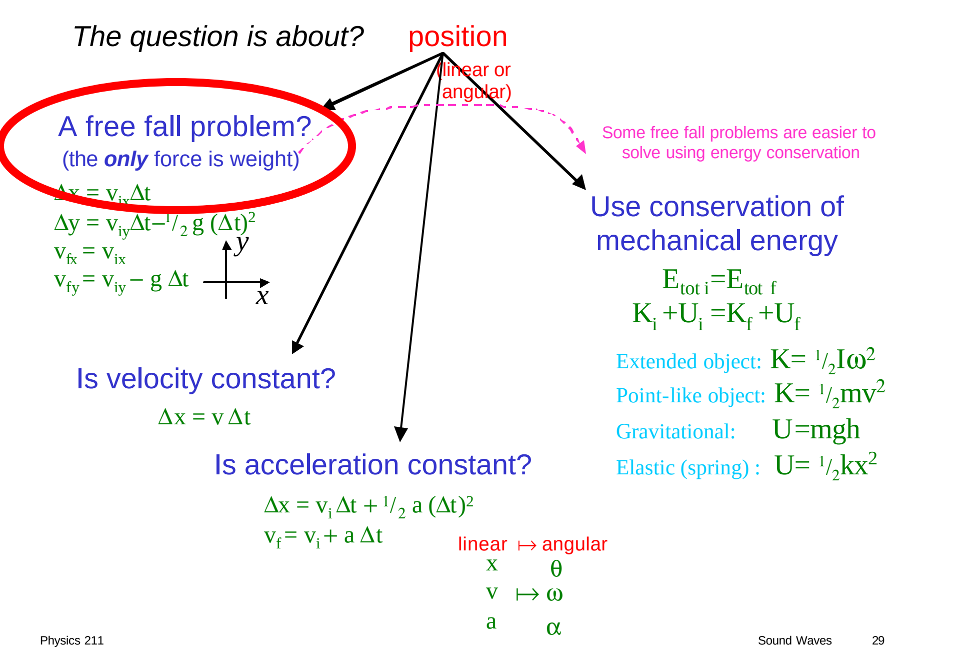
$$E_{tot\ f} - E_{tot\ i} = W_{ext.\ or\ non\text{-}cons.}$$

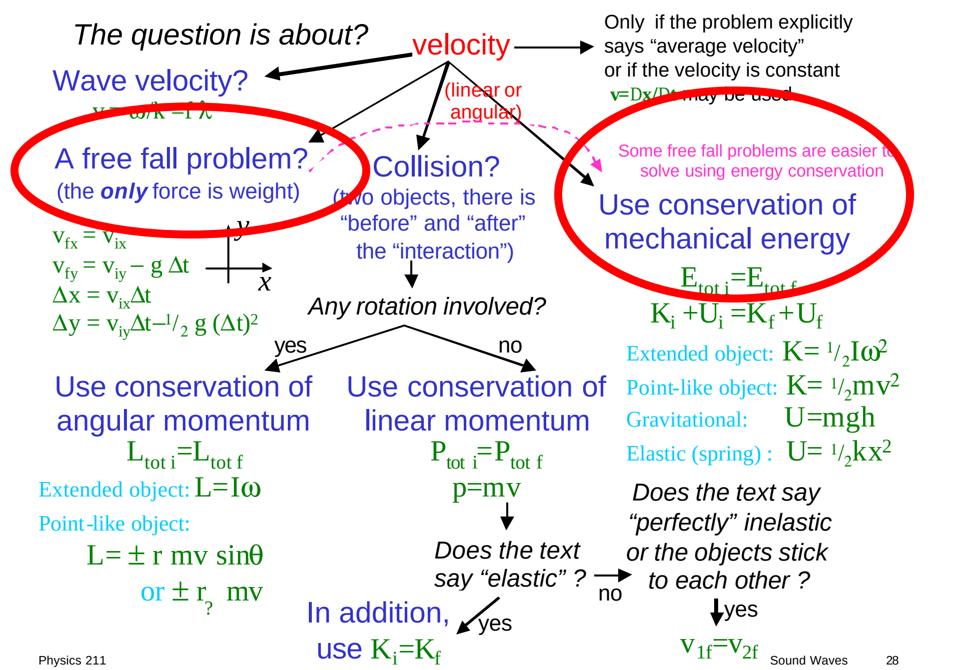
1. [15pts total] A diver runs off the diving board located at h=2m above the water with initial velocity v_i=3 m/s directed horizontally.



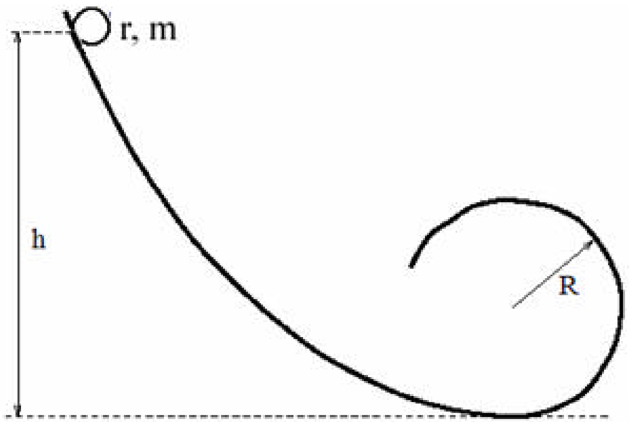
(1a) [10pts] How far does she fly in horizontal direction, Δx, before entering the water?

(1b) [5pts] What is her speed (i.e. magnitude of total velocity) v_f when she enters the water?



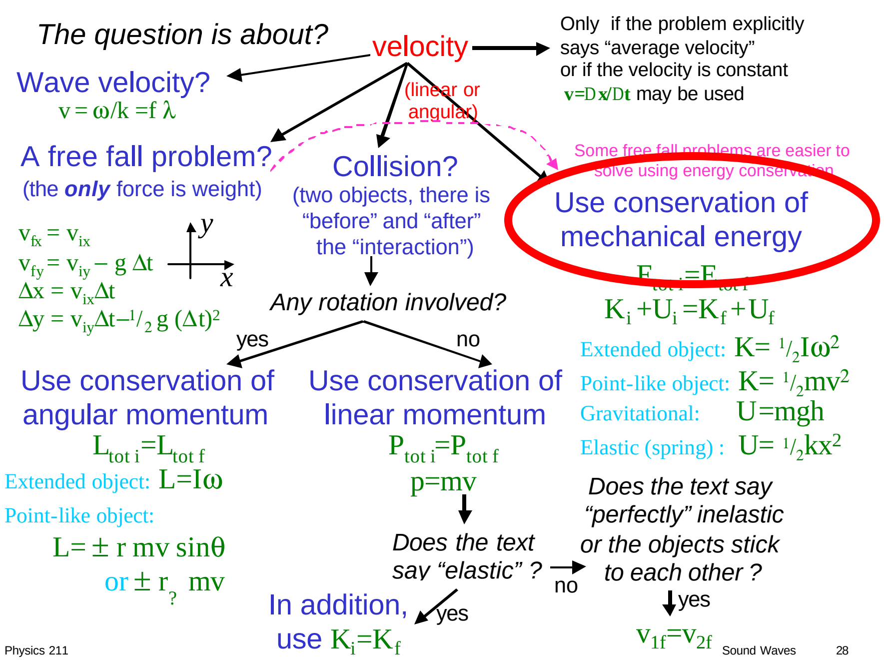


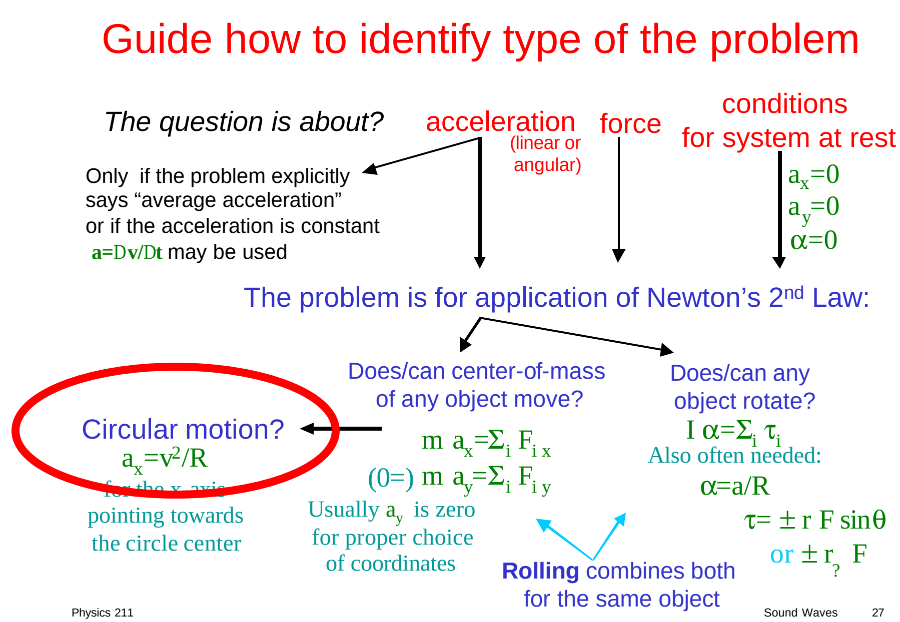
2. A uniform ball of mass r =0.1 m and mass m=3 kg rolls down without slipping along loop-the-loop track shown below. The radius of the loop is R=1.6 m. The ball is released from rest with its center at the height h = 12 m above the bottom of the track.



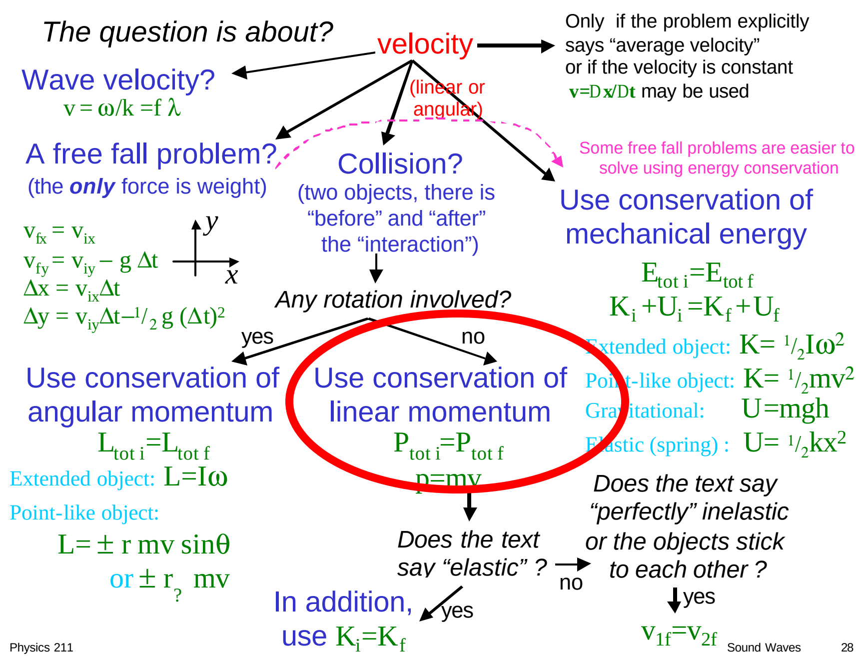
2b. What is the magnitude of the normal force exerted by the track on the ball at the top of the loop? (g=10m/s^2)

2a. What is the speed of center-of-mass of the ball at the top of the loop? (g=10m/s^2)

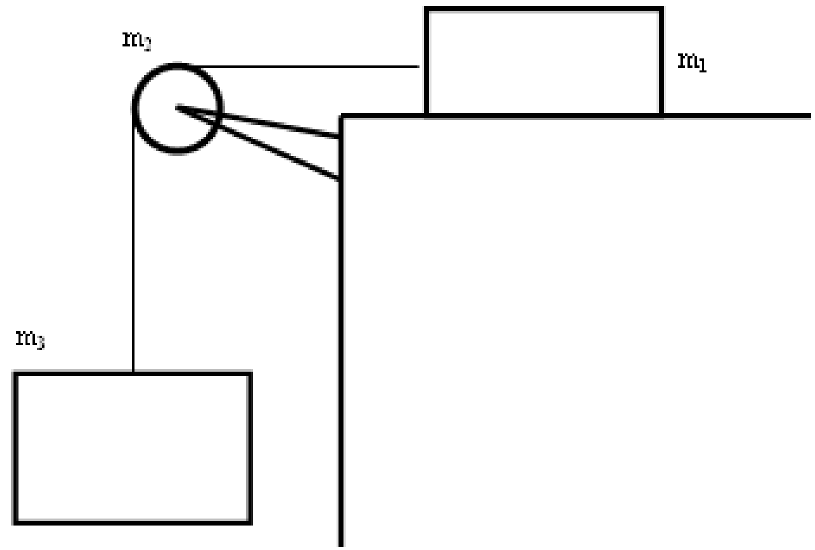




3. [10pts] A bullet is shot through a wooden block. The bullet has a mass of 0.003kg and its initial speed is 400 m/s. The block is initially at rest and has a mass of 5kg. The block has a speed of 5 m/s right after the bullet went through. Calculate the speed of the bullet after it went through the block.



4. Two blocks with masses $m_1=7$ kg and $m_3=5$ kg are connected by a massless string via pulley with a mass of $m_2=6$ kg. Assume the pulley is a uniform disk and that it rotates without a friction on its axle. The string is non-stretchable and doesn't slip on the pulley. Coefficient of kinetic friction for the block on the horizontal surface is $\mu=0.06$. Find acceleration of this block assuming it is moving to the left (use $g=10$ m/s^2).



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Rolling combines both for the same object

Circular motion?

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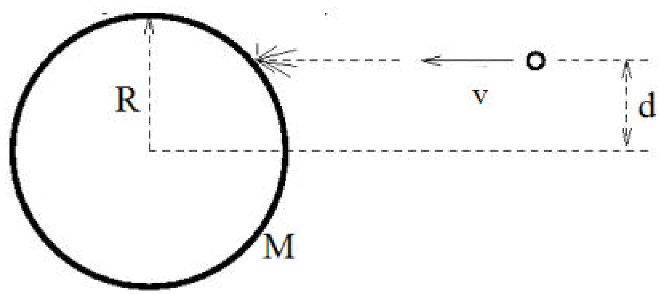
for the x-axis pointing towards the circle center

5. [10pts] Phobes is a small moon of Mars. It has a mass of M=5.8 10^{15} kg and a radius of R=7.5 10^3 m. For the purpose of the following problem, assume that Phobes has the shape of a uniform sphere and that it is initially at rest. Suppose a meteorite strikes Phobes at distance d=5 10^3 m off center and embeds itself inside Phobes, close to its surface. If the meteorite mass was m=3 10^8 kg and its speed was v=10^5 m/s, what is the angular velocity ω of Phobes about its axis of rotation after the collision?



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y
x

Collision?
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Use conservation of mechanical energy

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Extended object: $K = {}^1/_2 I\omega^2$
Point-like object: $K = {}^1/_2 mv^2$
Gravitational: $U = mgh$
Elastic (spring) : $U = {}^1/_2 kx^2$

Any rotation involved?

yes

no

Use conservation of angular momentum
$L_{tot\,i} = L_{tot\,f}$
Extended object: $L = I\omega$
Point-like object:
$L = \pm\, r\, mv \sin\theta$
or $\pm\, r_?\ mv$

Use conservation of linear momentum
$P_{tot\,i} = P_{tot\,f}$
$p = mv$

Does the text say "elastic" ?

no

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6. [10pts] A block of mass m_1=3kg is suspended from the end of uniform horizontal beam of length L=7m and mass m_2=5kg pinned to the wall at the other end (i.e. it is attached to the wall using a hinge). The beam is suspended on a cable attached to its end creating an angle of θ=35° with the beam (see below). What are the horizontal (R_x) and vertical (R_y) components of the reaction force exerted by the pin on the beam?